Factoring the Sum and Difference of Cubes

The Sum of Cubes Pattern:

$$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$$

In some problems it is easy to recognize the sum of cubes pattern. In this first example the “a” part is 3x and the “b” part is 2y.

$$a^3 + b^3 = (a + b)(a^2 - a \cdot b + b^2)$$
$$(3x)^3 + (2y)^3 = (3x + 2y)((3x)^2 - 3x \cdot 2y + (2y)^2)$$
$$(3x)^3 + (2y)^3 = (3x + 2y)(9x^2 - 6xy + 4y^2)$$

In this next example you need to recognize that 8 is the same as 2 cubed ($2 \bullet 2 \bullet 2 = 2^3 = 8$) or you won’t see that this is a sum of cubes problem. In this example the “a” part is 2 and the “b” part is y.

$$a^3 + b^3 = (a + b)(a^2 - a \cdot b + b^2)$$
$$8 + y^3 = (2 + y)(2^2 - 2 \cdot y + y^2)$$
$$2^3 + y^3 = (2 + y)(4 - 2y + y^2)$$

In this last example notice that 27 is the same as 3 cubed ($3 \bullet 3 \bullet 3 = 3^3 = 27$) and x^6 is $(x^2)^3$ or you won’t see that this is a sum of cubes problem. In this example the “a” part is 3 and the “b” part is x^2.

$$a^3 + b^3 = (a + b)(a^2 - a \cdot b + b^2)$$
$$27 + x^6 = (3 + x^2)(3^2 - 3 \cdot x^2 + (x^2)^2)$$
$$3^3 + (x^2)^3 = (3 + x^2)(9 - 3x^2 + x^4)$$

Since $(x^2)^3 = (x^3)^2$ why use $(x^2)^3$ instead of $(x^3)^2$ to replace x^6? Because x^6 has to be written as something cubed to fit the sum of cubes pattern. Writing it as $(x^3)^2$ does not fit the pattern, it makes it something squared.

The Difference of Cubes Pattern:

$$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$$

In some problems it is easy to recognize the difference of cubes pattern. In this example the “a” part is 3x and the “b” part is 2y.

$$a^3 - b^3 = (a - b)(a^2 + a \cdot b + b^2)$$
$$(3x)^3 - (2y)^3 = (3x - 2y)((3x)^2 + 3x \cdot 2y + (2y)^2)$$
$$(3x)^3 - (2y)^3 = (3x - 2y)(9x^2 + 6xy + 4y^2)$$

For this example you need to recognize that 8 is the same as 2 cubed ($2 \bullet 2 \bullet 2 = 2^3 = 8$) or you won’t see that this is a difference of cubes problem. In this example the “a” part is 2 and the “b” part is y.

$$a^3 - b^3 = (a - b)(a^2 + a \cdot b + b^2)$$
$$8 - y^3 = (2 - y)(2^2 + 2 \cdot y + y^2)$$
$$2^3 - y^3 = (2 - y)(4 + 2y + y^2)$$

In this final example notice that 27 is the same as 3 cubed ($3 \bullet 3 \bullet 3 = 3^3 = 27$) and x^6 is $(x^2)^3$ or you won’t see that this is a difference of cubes problem. In this example the “a” part is 3 and the “b” part is x^2.

$$a^3 - b^3 = (a - b)(a^2 + a \cdot b + b^2)$$
$$27 - x^6 = (3 - x^2)(3^2 + 3 \cdot x^2 + (x^2)^2)$$
$$3^3 - (x^2)^3 = (3 - x^2)(9 + 3x^2 + x^4)$$