Transformation of Functions

When a function is transformed, several changes to its graph are possible. The graph can shift right / left / up / or down. It can stretch or compress vertically or horizontally. It can also be reflected horizontally across the y-axis (the line x = 0) or reflected vertically across the x-axis (the line y = 0).

It is allowable to apply two or more transformations to a function at the same time. When applying more than one transformation, the order they are applied does make a difference in some situations and no difference in others. Take $f(x) = x^2$ for example. If you flip it first and then add 2 ($y = -x^2 + 2$) you will not get the same graph as adding 2 first and then flipping it ($y = -(x^2 + 2)$. Be sure to follow the order of operations when evaluating the transformed function to get the correct graph.

Follow this link:
https://www.desmos.com/calculator
to graph all 3 functions above on the same graph and you will see how the transformations are different. Use the order of operations when evaluating a function for its x-values to be sure to get the correct y-values.

Vertical Transformations are the result of modifying the output (y)

To shift the graph up or down:

- To shift the graph up, add a non-negative, non-zero number *a* to the output f(x). For example to move $f(x) = x^2$ up five units you add 5 to the function to get the transformed function $g(x) = x^2 + a$ which becomes $g(x) = x^2 + 5$.

- To shift the graph down, subtract a non-negative, non-zero number *a* from the original function to get the transformed function. For example to move $f(x) = x^3$ down 2 units, subtract 2 from the original function. $f(x) – 2$ becomes $g(x) = x^3 – 2$.

To stretch or compress the graph vertically:
Multiply the output (y) by a non-zero, non-negative constant *a*.

- To stretch (expand) the graph vertically:
  If *a* >1, each point on the original graph will stretch vertically from the line y = 0 (the x-axis) by the factor *a*. If for example a point was 3 units above

the x-axis on the original graph and the graph was stretched vertically by a factor of 2, the corresponding point of the vertically stretched graph would be twice the vertical distance from the original point which would be 6 units above the x-axis. If a point on the original graph is 4 units below the x-axis, the transformed point will be 8 units below the x-axis. For this example take the y-value of any point on the original graph and multiply it by 2 and you will get the corresponding y-value of the transformed point.

- To compress (shrink) the graph vertically:
  If $0 < a < 1$, each point on the original graph will compress vertically from the line y = 0 (the x-axis) by the factor a. If for example a point was 10 units above the x-axis on the original graph and the graph was compressed vertically by a factor of 1/2, the corresponding point of the vertically compressed graph would be half the vertical distance from the original point which would be 5 units above the x-axis. If a point on the original graph is 12 units below the x-axis, the transformed point will be 6 units below the x-axis. For this example take the y-value of any point on the original graph and multiply it by 1/2 and you will get the corresponding y-value of the transformed point.

To reflect over the x-axis (the line y = 0):

- Multiply the whole function by -1.

Horizontal Transformations are the result of modifying the input x

This is the easy way to think about it:
Just do the opposite of what you think you should do (except when flipping the graph over the y-axis, in this case just multiply x by -1).

To shift the graph left or right horizontally:
To shift the graph left 3 units, you would think that you need to subtract 3 from x but that is not what you need to do. Do the opposite, you have to add 3 to x. To shift the graph to the right 4 units you would think that you have to add 4 to x but do the opposite instead. You have to subtract 4 from x to move the graph right 4 units.

To compress or stretch the graph horizontally:
To stretch it by 5/3 you multiply x by the reciprocal which is 3/5. To compress it by 1/2 you multiply x by 2 which is the reciprocal of 1/2.

To reflect (flip) the graph over the y-axis (the line x=0):
Multiply x by -1.

This is the mathematical explanation of horizontally shifting, stretching, compressing, and reflecting graphs (it may seem confusing).

To shift the graph right or left:
Put the function into this pattern:
(x - h). The value of h is what you are adjusting the value of x with.

- If h is positive the graph moves to the right h units. So (x - 4) would move the graph 4 units to the right. For example, shifting $y = x^2$ four units right would look like this: $y = (x – 4)^2$.

- If h is negative the graph moves left. For example, (x + 3) would be rewritten in the correct pattern as (x - -3). h must be after the first minus sign. In this case h is negative and would move the graph to the left 3 units. For example, shifting $y = x^2$ three units left would look like this: $y = (x - - 3)^2$ which simplifies to $y = (x + 3)^2$.

To stretch or compress the graph horizontally: Multiply x by a non-negative, non-zero number 1/a. If a is not a fraction, make it a fraction by putting it over 1.

- To stretch (expand) a graph horizontally away from the y-axis, *a* must be greater than 1 which makes the reciprocal of a, 1/a, less than 1. You need to multiply x in the function by the reciprocal of *a* which is *1/a*. The graph will stretch (expand) horizontally away from the y-axis (the line x = 0) by the factor *a*. For example, if *a* = 2, (*1/a* = 1/2) you are stretching the graph by a factor of 2. The x-coordinate of each point on the graph will be multiplied by 2 to get the x-coordinate of the transformed point. Consider transforming the function $y = x^2$ by doubling the horizontal distance of each point from the y-axis to get the transformed function $y = ((1/2)x)^2$. On the original function, the point (-2, 4) has a corresponding transformed point (-4, 4) which is twice the distance horizontally from its original point. The point (-1, 1) on the original function has a corresponding point (-2, 1) which is twice the horizontal distance from the y-axis as the original point. Additionally the point (2, 4) on the original function transforms to (4, 4) on the transformed function (twice the horizontal distance as the original point).

- To compress (shrink) a graph horizontally towards the y-axis, *a* must be less than 1. The graph will compress (shrink) horizontally towards the y-axis (the line x = 0) by the factor *a*. For example, if *a* = 1/3 (*1/a* = 3) then the graph will compress (shrink) horizontally towards the y-axis by a factor of 1/3. Consider transforming the function $y = x^2$ by compressing the horizontal distance of each point from the y-axis by 1/3 to get the transformed function $y = (3x)^2$. On the original function the point (-6, 36) has a corresponding transformed point (-2, 36) which is one-third the distance horizontally from the y-axis than the original point is. The point (-3, 9) on the original function has a corresponding point (-1, 9) which is one-third the horizontal distance from the y-axis as the original point. Additionally the point (3, 9) on the original function transforms to (1, 9) on the transformed function (one-third the horizontal distance from the y-axis as the original point).

To reflect the graph over the y-axis: Multiply x by -1. The graph will flip (reflect) over the y-axis but it won't stretch or compress.