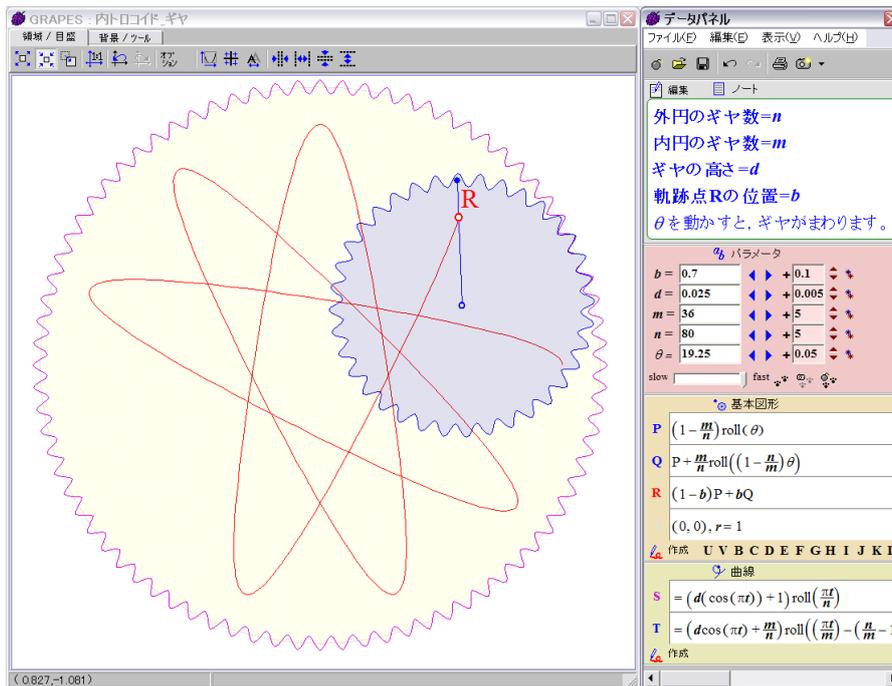


GRAPES USER'S MANUAL

GRAPES Tentative Ver 6.7



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Users Manual

Operating Environment, Copyright, and Latest Version

- Operating Environment

- a. Operating Systems

- WindowsNT/2000/XP/Vista

- Operation under Windows95/98/Me has not been confirmed. Use of Ver 6.16 or earlier is recommended under these operating systems.

- b. Memory

- 256MB or more (512MB or more recommended).

- c. Monitor

- SVGA (800 x 600) or better

- 65,000 colors (16-bit color) or better

- d. Hard Disk Drive

- A floppy disk drive is sufficient if only GRAPES is required, however running from a floppy disk drive is not recommended.

- Copyright

- ◆ GRAPES is freeware and may be copied, distributed and used without restriction.
 - ◆ Notification of distribution for commercial use is appreciated.
 - ◆ Copyright for GRAPES and the associated manual is held by Katsuhisa Tomoda.
 - ◆ Copyright for data and images created using GRAPES is held by the creator.

- Cautions for Use

- ◆ The author of this software assumes no responsibility for any results or consequence of its use.
 - ◆ The author endeavours to eliminate bugs in the software, however this may not always be possible.

- Latest Version

- The latest version of the software is available at the following URL:

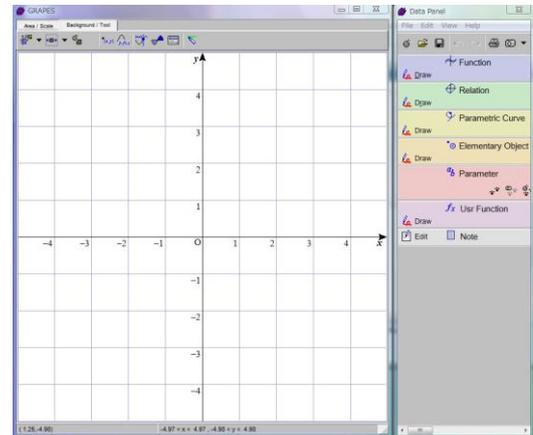
- <http://www.osaka-kyoiku.ac.jp/~tomodak/grapes/>

Chapter 1 GRAPES Basics

1-1 GRAPES Appearance

- Graph Window and Data Panel

The GRAPES screen is composed of two windows – the Graph Window and the Data Panel. The Graph Window is designed for the display of graphs. The Data Panel is designed for input of data which forms the basis of the graphs (e.g., function equations, parameter values).



- Graph Window

From the top of the screen, the Graph Window comprise of the Control Pallet, the Graph Display Area, and the Status Bar.

- ◆ Graph Area

The graph of the function in the Data Panel is displayed in the Graph Area.

The mouse is used to change the Area, move basic graphic elements (e.g. points), and move stickers, and to use pop-up menus with right-click.

- ◆ Graph Window Size

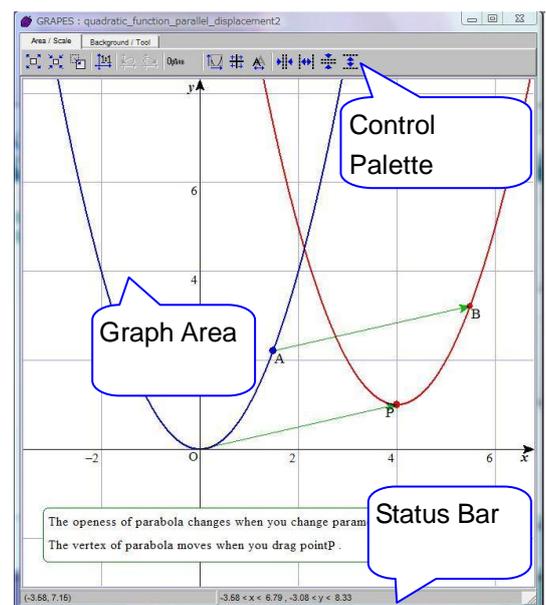
The size of the Graph Area is changed by dragging the corner or border of the Graph Window.

- ◆ Control Pallet

The Control Pallet contains the Area Pallet for setting the Display Area, the Scale Pallet, the Background Pallet, and the Tools Pallet with the tools for investigating the graph.

- ◆ Status Bar

The Status Bar at the bottom of the screen displays the cursor coordinates, and hints for the Graph Display Area and manipulation.



- Data Panel

- ◆ Menu Bar

Four menus ([File], [Edit], [View], [Help]) are available in the menu bar.

- ◆ Toolbar

The toolbar contains seven function buttons focused on file operations ([Initialize], [Open], [Save], [Undo], [Redo], [Print], [Capture]).

- ◆ FunctionArea

Handles function graphs displayed in the $y = f(x)$ format.

- ◆ RelationArea

Handles graphs and areas expressed by equality and inequality equations of x and y .

- ◆ Parametric CurveArea

Handles curves expressed by parameters, and graphs of polar equations.

- ◆ Elementary Object Area

Handles points, circles, horizontal and vertical lines.

- ◆ Parameter Area

Used for manipulation of parameters and images.

- ◆ User Function Area

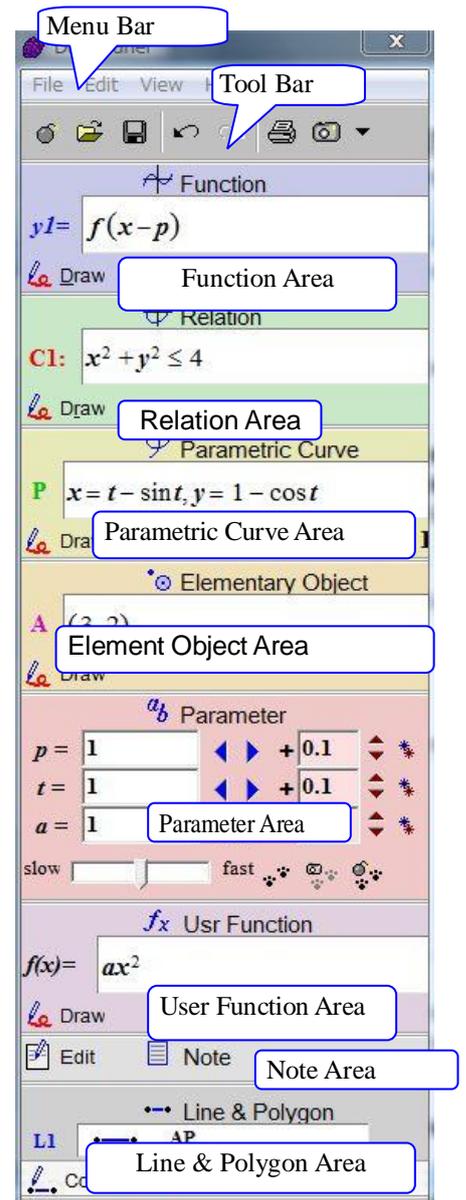
Used for entering an equation when defining a function.

- ◆ NoteArea

Used for editing stickers displaying explanations on the screen, scripts (a type of program), data tables, and text notes.

- ◆ Line & Polygon Area

Displays setup information for linked graphic elements (e.g., line segments, polygons).



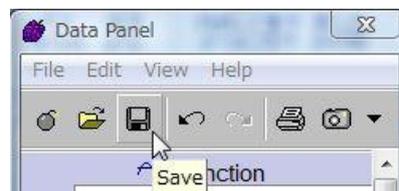
1-2 Saving and Loading Projects

- Projects

Data created with GRAPES is referred to as a 'project'. A project includes data created with GRAPES (e.g., graph equations, parameters, images, display areas, scale settings), and setup information.

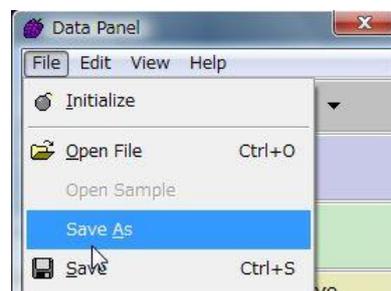
- Saving a New Project

Click on the  (Save) button on the Toolbar, or click on [Save] in the [File] menu.



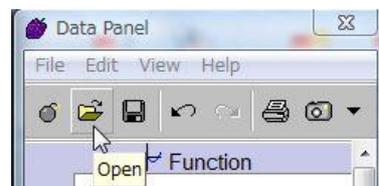
- Save Under Another Name

Click on [Save As] in the [File] menu.



- Load a Project (open file)

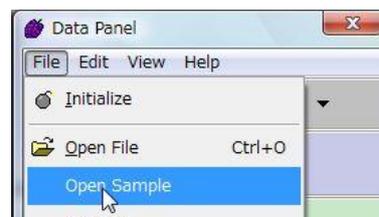
Click on the  (Open) button, or click on [Open] in the [File] menu.



- Open a Sample

Click on [Open Sample] in the [File] menu.

This menu is usable only if the GRAPES folder contains a Samples folder.



What is a GRAPES project file?

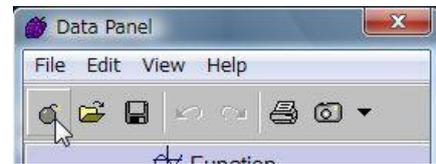
A GRAPES project is saved with the '.gps' identifier. A GRAPES project file includes all the data necessary to run GRAPES. GRAPES image data has the same file name as the project file, however it is saved with the '.gpp' identifier. Loss of the the image file will not present a problem if the project file is available.

Compatibility of project files is maintained when upgrading GRAPES to a new version, and existing data can therefore be used without problems.

1-3 Initializing a Project and Changing Default Values, and Associating Files

- Initializing of Project

Click on the  (Initialize) button, or click on [Initialize] in the [File] menu.



All data currently in preparation is deleted, and the system returns to the condition at startup.

Data already saved is not deleted.

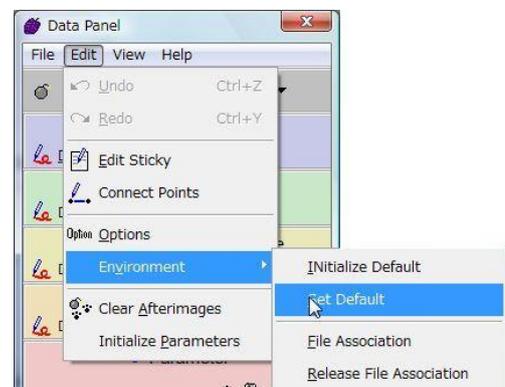
- Changing Default Values

All values are initialized to the default values at startup and at initialization.

1. Select values to be set as defaults.

Default data which may be changed includes most optional settings (e.g., settings related to display areas, window size, and scales).

2. Click on [Set Default] in [Environment] in the [Edit] menu.



- Initializing Default Values

Click on [Initialize Default] in [Environment] in the [Edit] menu.

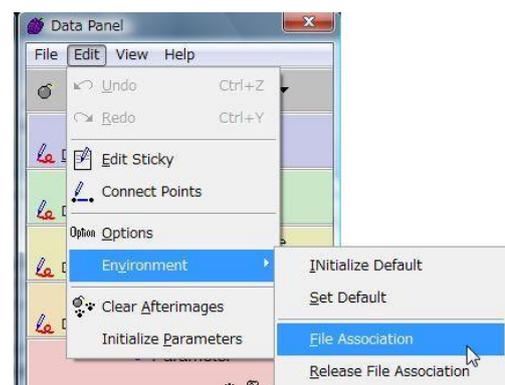
The defaults selected by the user are returned to their original values.

- Associating Files

If files have been **associated**, GRAPES may be started, and the file opened, simply by double-clicking on the GRAPES project file (*.gps').

Click on [File Association] in [Environment] in the [Edit] menu.

The GRAPES project file icon is changed to the 'bunch of grapes' icon. Note that this change in the icon may occur only after restart in some cases.



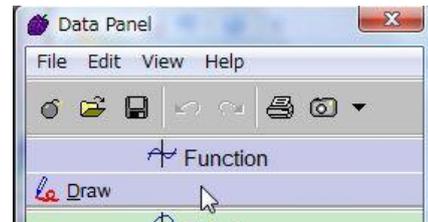
☆ Under Windows Vista, the operation described above is accomplished by first closing GRAPES, right-clicking on the icon, selecting [Run as Administrator], and then restarting.

Chapter 2 Function Graphs

2-1 Creating and Adding Function Graphs

- Adding and Creating Function Graphs

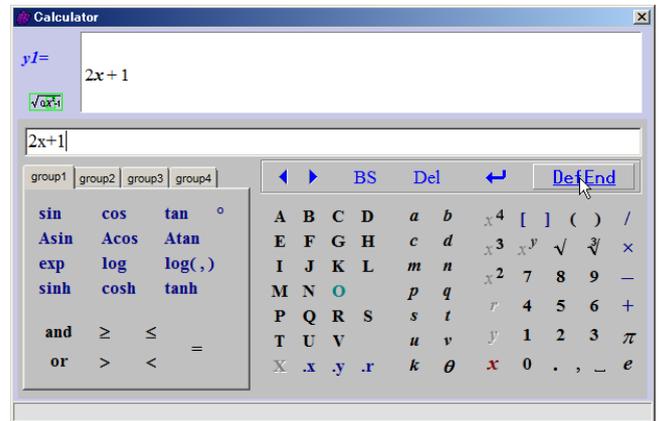
1. Click on [Draw] in the Function Area.



2. Enter the equation with the scientific calculator.

Enter from the keyboard or with the buttons on the scientific calculator.

When input is complete, click on [DefEnd].

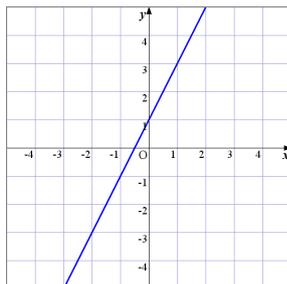
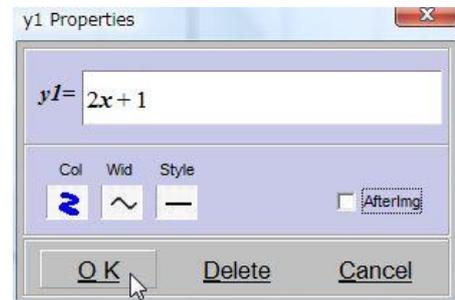


3. Properties of the graph style.

The Function Graph Property Window is displayed.

See the following page for details of setting the graph color and thickness.

When all the settings have been entered, click on [OK].



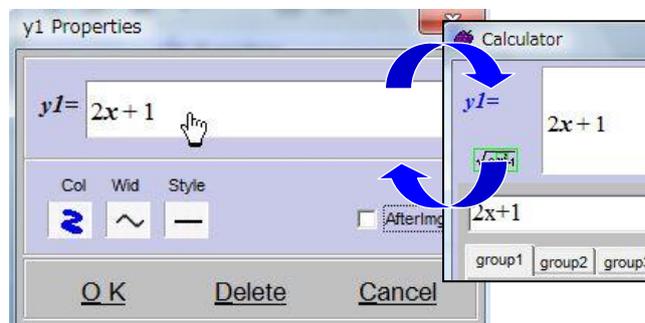
Up to 20 function graphs may be drawn.

2-2 Function Graph Properties

Function equations, graph color and thickness, line type, and image settings are entered in the Function Graph Property Window.

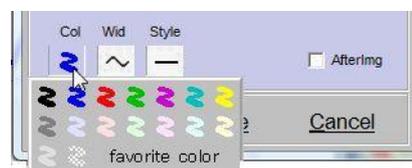
- Changing Equations

1. Click on the Function Equation Window.
2. Change the equation with the scientific calculator.



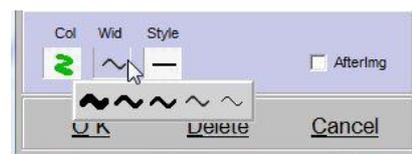
- Changing the Graph Color

1. Point to the Graph Color Window.
2. Select the desired color from the palette.



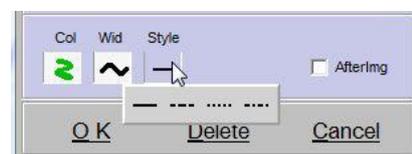
- Changing the Graph Thickness

1. Point to the Graph Thickness Window.
2. Select the desired thickness from the palette.



- Changing the Graph Line Type

1. Point to the Graph Line Type Window.
2. Select the desired line type from the palette.



- Changing the Image Settings

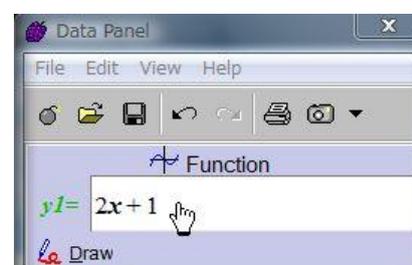
Click on [AfterImg].



2-3 Changing and Deleting Function Graphs

- Changing Function Graphs

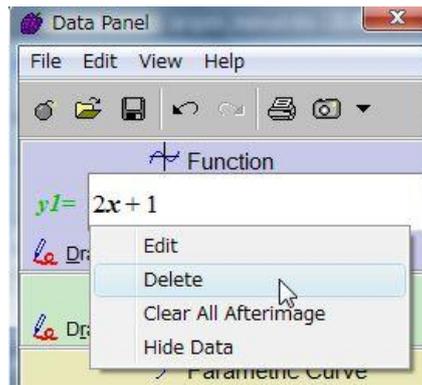
1. Click on the Function Equation Window in the Function Area.
2. Make the changes in the Function Properties Window.



The graph equation, color, thickness, and whether or not an image is required, may be changed.

- Deleting Function Graphs

1. Right-click on the Function Equation Window in the Function Area.
2. Select [Delete] from the pop-up menu.
3. Click on [OK] to confirm delete.



2-4 Moving the Graph

- 14 Parameters

The following 14 letters may be assigned to parameters.

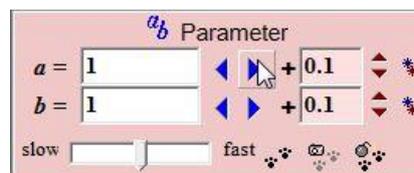
$$a, b, c, d, k, m, n, p, q, s, t, u, v, \theta$$

A parameter is displayed if it is appropriate to change its value.

- Increasing and Decreasing Parameter Values

Click on the   (increase and decrease) buttons.

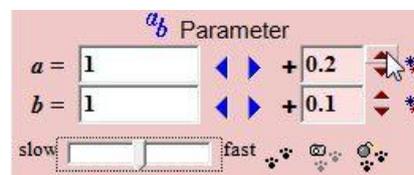
☆ The value changes continuously when a button is held down.



- Range of Increase/Decrease

Click on the   (range of increase/decrease) buttons.

☆ Click on the Range of Increase/Decrease Window to enter the value from the keyboard.



- Changing the Rate of Increase/Decrease

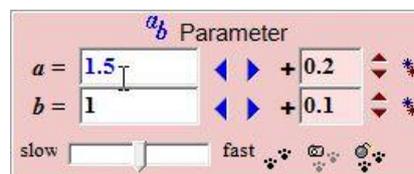
Move the slider to change the rate of increase/decrease.

Changes the rate at which the value is changed when the increase/decrease buttons is held down.

- Substituting Values in Parameters

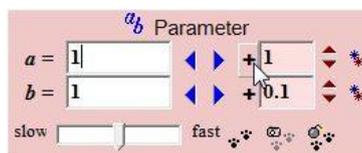
Click on the Parameter Window to enter the value from the keyboard.

☆ Double-click to use the Scientific Calculator.



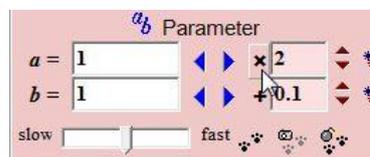
- Changing the Method of Increase/Decrease

Click on [+].



Parameter changes 1->2->3->4

changes 1->2->4->8



Parameter

- Synchronizing Parameters

It is possible to synchronize parameters so that changing one parameter changes other parameters.

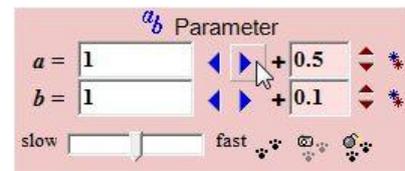
1. Click on the  (synchronize) button for all the parameters

to be synchronized.

2. Increasing and Decreasing Parameter Values

Values of other linked parameters are increased/decreased.

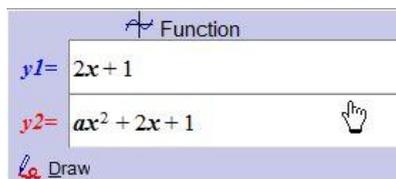
In the example at right, increasing/decreasing value a increases/decreases value b .



2-5 Leaving a Graph – the Afterimage

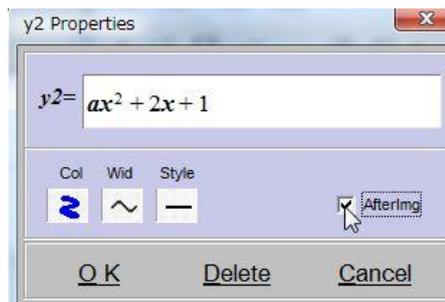
The graph can be changed by increasing/decreasing the parameter values. The Afterimage is used to leave a record of the original graph.

- Leaving an afterimage



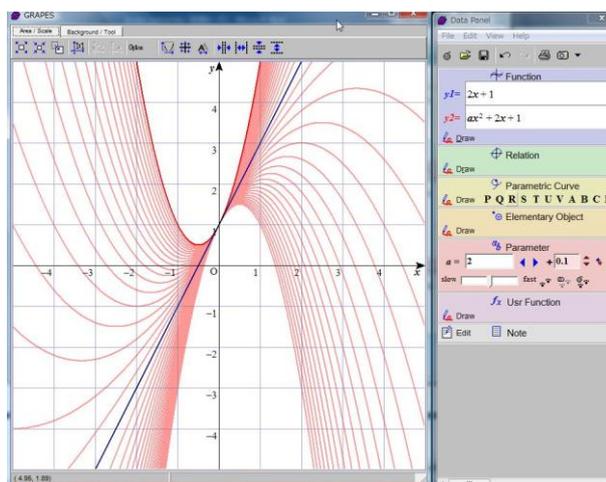
1. Click on the graph of the function equation for which the afterimage is required.

2. Click on [Afterimg].



3. Click on [OK].

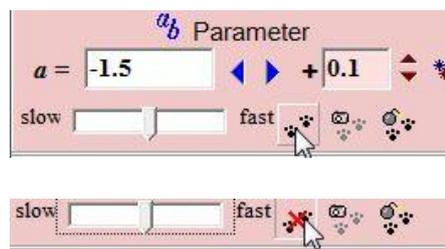
4. Change the parameter.



- Temporarily Halt Afterimage Recording

Click on the  button (Parameter Area Afterimage OFF).

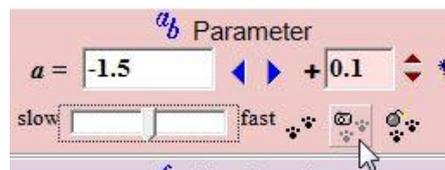
The button will remain pressed, and the Afterimage will not be recorded. Click again to clear.



- Create Afterimage

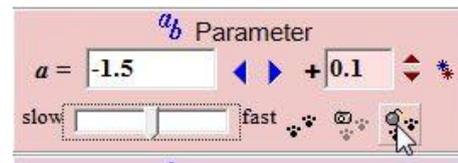
Click on the  button (Create Afterimage of Parameter Area).

It is possible to record Afterimages of all currently drawn graphs.



- Deleting the Afterimage

Click on the  button (Delete Afterimage of Parameter Area).



Chapter 3 Display Area and Scale Adjustment

3-1 Enlarging the Graph

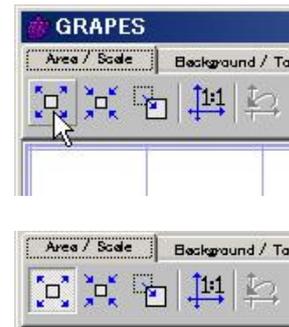
- Changes to the Area - Enlarging the Graph

1. Click on the  button (Zoom in).

The Zoom in button is selected.

This status is referred to as as the Enlarge Mode.

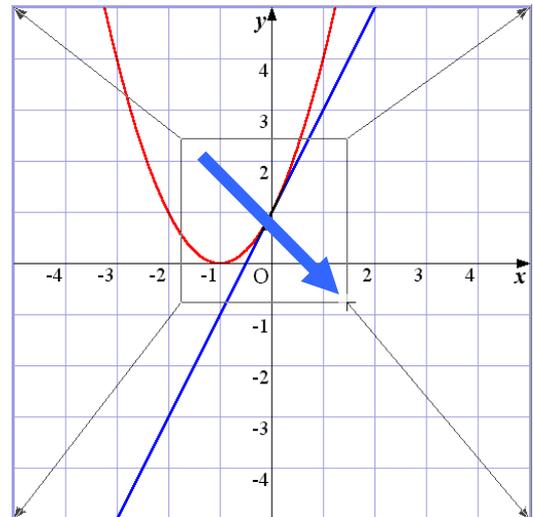
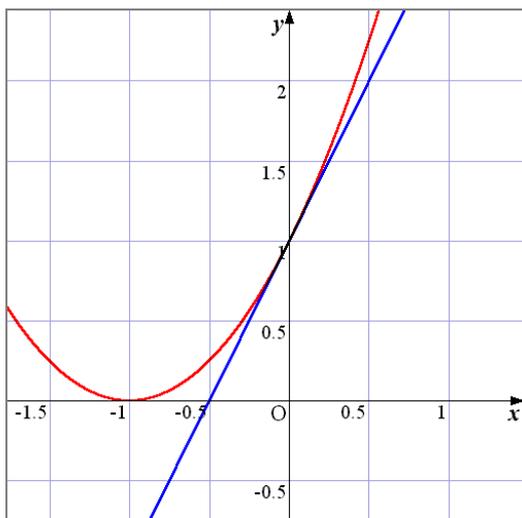
Click again to clear the Enlarge Mode.



2. Specify a rectangular area.

At one end of a diagonal in the rectangular area, and

- a. left-click,
- b. drag while holding the left button down,
- c. and then, release at the other end of the diagonal.



- The aspect ratio is fixed when enlarging. Change the aspect ratio with Shift key + drag.
- Use the Ctrl key + drag to enlarge with the point at which the mouse is clicked at the center.
- Double-click on the center of the area to be enlarged to double the size of the graph.

3-2 Shrinking the Graph

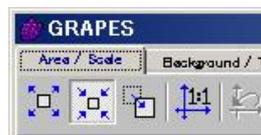
● Changes to the Area – Shrinking the Graph

1. Click on the  button (Zoom out).

The Zoom out button is selected.

This status is referred to as as the Shrink Mode.

Click again to clear the Shrink Mode.



2. Specify a rectangular area.

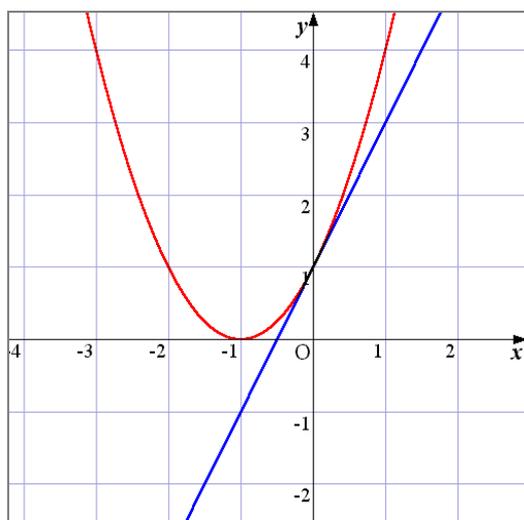
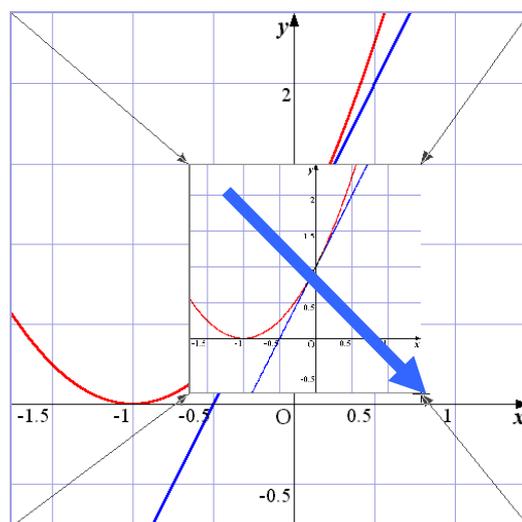
The graph is shrunk into the specified rectangular area.

At one end of a diagonal in the rectangular area, and

- a. left-click,
- b. drag while holding the left button down,
- c. and then, release at the other end of the diagonal.

The reduced image of the graph is displayed on the screen.

Use it for reference.



- The aspect ratio is fixed when shrinking. Change the aspect ratio with Shift key + drag.
- Use the Ctrl key + drag to enlarge and shrink with the point at which the mouse is clicked at the center.
- Double-click on the center of the area to be shrunk to halve the size of the graph.

3-3 Moving the Graph

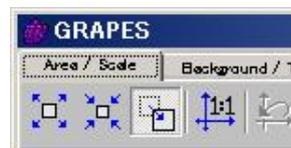
- Changes to the Area – Moving the Graph

1. Click on the  button (Move).

The Move button is selected.

This status is referred to as the Move Mode.

Click again to clear the Move Mode.



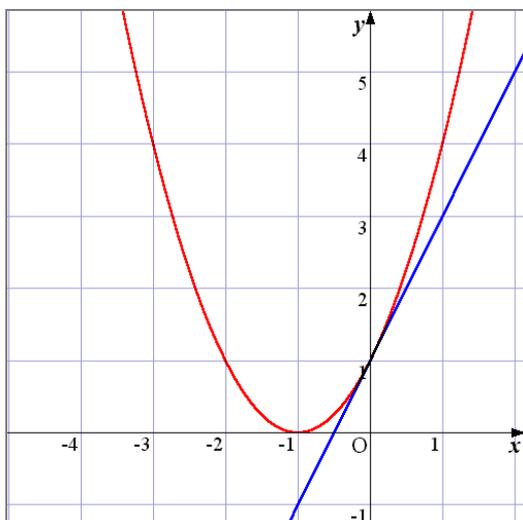
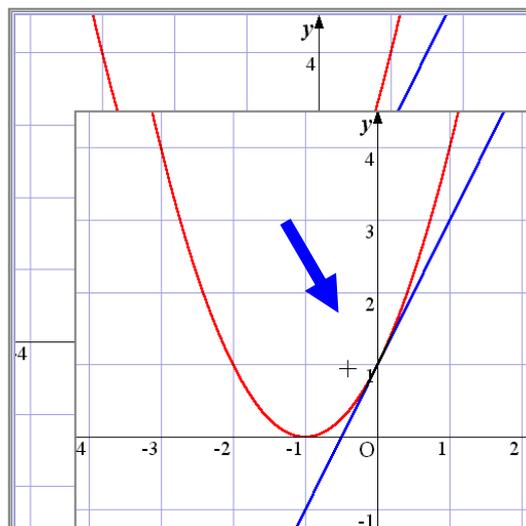
2. Specify a rectangular area.

Anywhere in the Graph Window,

- left-click,
- drag while holding the left button down,
- and then release at the other end of the diagonal.

A copy of the graph moves.

Determine the amount of movement required.



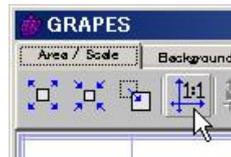
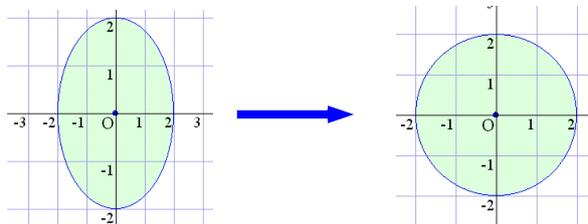
- If using a mouse with a scroll wheel, push the wheel and drag to move the graph. This eliminates the need to select the Move Mode.

3-4 Changes to the Area – 1:1, Undo, Redo

- Fix the aspect ratio to 1:1

Click on the  button (1:1).

Changes the aspect ratio of the graph to 1-to-1.

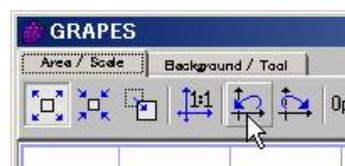


- Undo a Change to the Area

Click on the  button (Undo of Domain).

Returns to the immediate previous display.

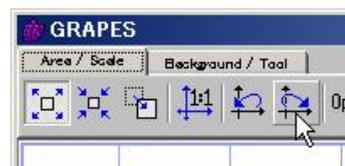
Up to 50 consecutive Undo operations are possible.



- Redo a Change to the Area

Click on the  button (Redo of Domain).

Cancels an Undo operation.

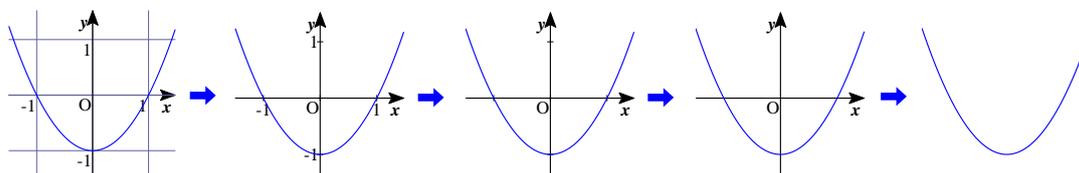


3-5 Displaying Scales and Axes

- Switch Between Scales and Coordinates Display

Click on the  button (Graduations).

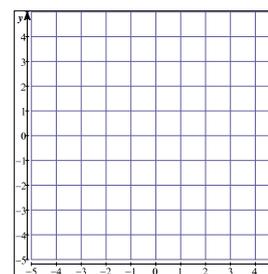
Switches between a scale and coordinates.



- Displaying the Axes Outside the Area

Click on the  button (Out Area Graduations).

Click again to clear the display.



3-6 Changing Scale Ranges

- Changing Scale Ranges



Click on the  button (Shrink x Axis Scale Range) to shrink the range of the scale on the x axis.

Click on the  button (Expand x Axis Scale Range) to expand the range of the scale on the x axis.

Click on the  button (Shrink y Axis Scale Range) to shrink the range of the scale on the y axis.

Click on the  button (Expand y Axis Scale Range) to expand the range of the scale on the y axis.

The scale range changes as follows:

$$1 \rightarrow \times 2 \rightarrow \times 5 \rightarrow \times 10$$

$$1 \rightarrow \times \frac{1}{2} \rightarrow \times \frac{1}{5} \rightarrow \times \frac{1}{10}$$

Note that the scale range changes as follows when the base values are π , 2π , 90 and 180 etc.

$$1 \rightarrow \times 2 \rightarrow \times 3 \rightarrow \times 6 \rightarrow \times 9 \rightarrow \times 18 \rightarrow \times 45 \rightarrow \times 90$$

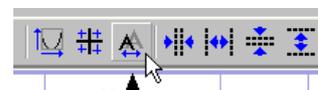
$$1 \rightarrow \times \frac{1}{2} \rightarrow \times \frac{1}{3} \rightarrow \times \frac{1}{6} \rightarrow \times \frac{1}{9} \rightarrow \times \frac{1}{18} \rightarrow \times \frac{1}{45} \rightarrow \times \frac{1}{90}$$

- Changing the Spacing of the Scale Notation

Click on    or  while holding down the Ctrl key.

- Changing the Size of Scale and Label Notation

Click on the  button (Change Scale Notation Size) on the Scale Pallet.

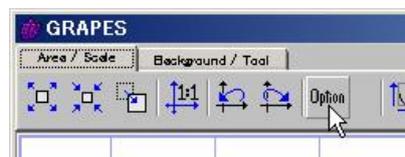


3-7 Standard Settings for Scales and Areas

- Displaying the Options Window

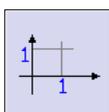
Click on the  button (Area Pallet Options).

The Options Window is displayed.

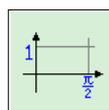


- Using the Standard Settings

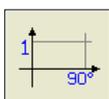
Click on one of the four settings displayed to select it.



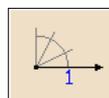
Set angles in radians, and 1 as the standard value for x and y axis scales. This is the default settings for GRAPES.



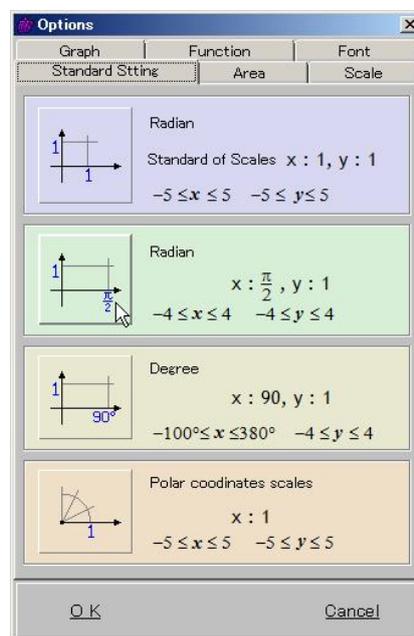
Set angles in radians, and $\frac{\pi}{2}$ as the standard value for the x axis scale. Select when drawing graphs of trigonometric functions with the circular method.



Set angles entered with the frequency method, and 90 set as the standard value for the x axis scale. ‘°’ added to the angle on the x axis scale. Select when drawing graphs of trigonometric functions with the frequency method.



Displays a polar coordinate scale.



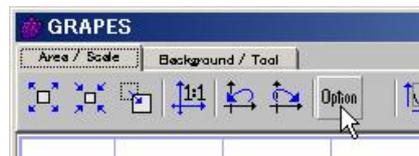
- Completing Setup

Click on [OK].

3-8 Detailed Settings for Scales

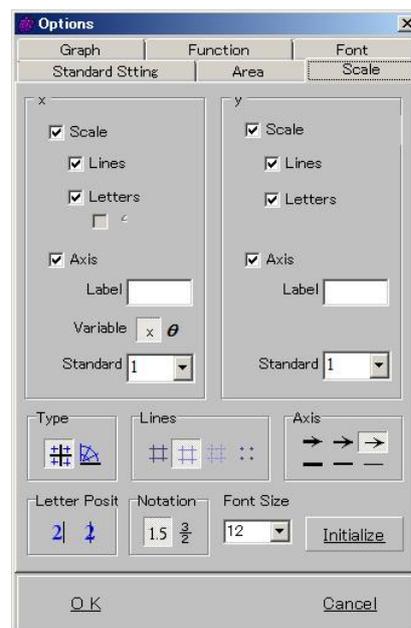
- Displaying the Scale Setup Window

1. Click on the  button (Area Pallet Options).
2. Click on the [Scale] tab.



- Setting Up Scale Displays

- Clear the check from [Lines] to remove grid lines.
 Clear the check from [Letters] to remove scale notation.



- Setting Up Axis Displays

Clear the check from [Axis] to remove the axis.

The letters is also removed.

Overwrite the letters in the Label Window to change the axis label.

'x' and 'y' are displayed by default as the axis labels, however these may be overwritten with any desired labels.

Click on  or  to change the x axis variable.

If θ is selected as the axis variable, θ is handled as an independent variable.

Used for displaying graphs such as $y = \sin \theta$.

- Setting Up Standard Values for Scales

Enter the standard value in the Standard Value Window.

The scale width normally changes as follows based on this value.

$$1 \rightarrow \times 2 \rightarrow \times 5 \rightarrow \times 10$$

- Setting Up Letters

Select the position for the letters to be displayed.

If  is selected, the letters is displayed between grid lines.

If  is selected, the letters is displayed over grid lines.

- Fractional Letters

If  is selected, the letters is displayed as fractions.

3-9 Detailed Settings for Areas

- Displaying the Area Setup Window

Click on the [Area] tab in the Options Window.

- Setting Screen Size

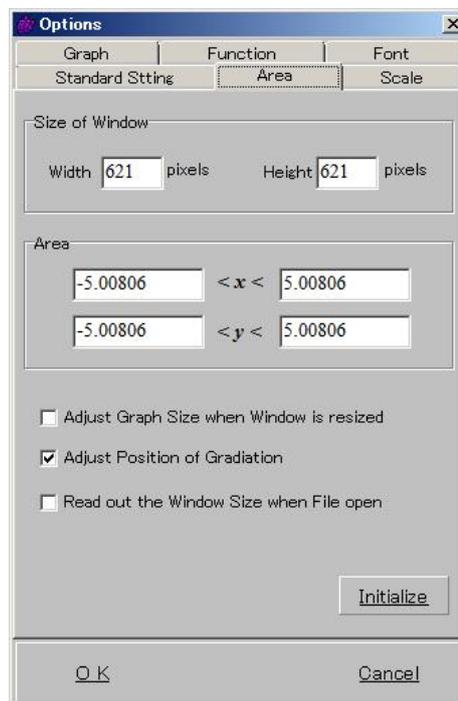
Enter the width and height.

Screen size is the size of the Graph Display Area.

Specify screen size in integer values between 150 x 150 and 1600 x 1600.

- Setting the Drawing Area

Enter the x and y ranges.



- Setting Options When Changing Window Size

By default, the graph size remains unchanged and the display area changes, when window size is changed.

If a check is placed in the [Adjust Graph Size when Window is resized] checkbox, the graph size also changes when the window size is changed.

- Aligning the Position of the Scale With Pixels on the Screen

Adjusts the area so that the scale grid lines are aligned accurately with the pixels on the screen when the scale value is an integer.

- Read Out the Window Size When File Open

As to the screen size when opening a file, only the aspect ratio will be reproduced, not the screen size at the time of creating the file. If you give this a check, the original screen size when creating a file will be reproduced.

- Returning to the Status at Startup

Click on [Initialize].

Returns to the screen size and drawing area size at startup.

- Completing Setup

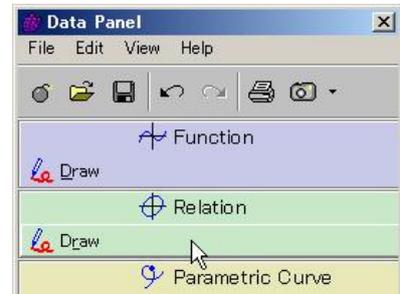
Click on [OK].

Chapter 4 Relation Graphs

4-1 Creating and Adding Relation Graphs

- Creating and Adding Relation Graphs

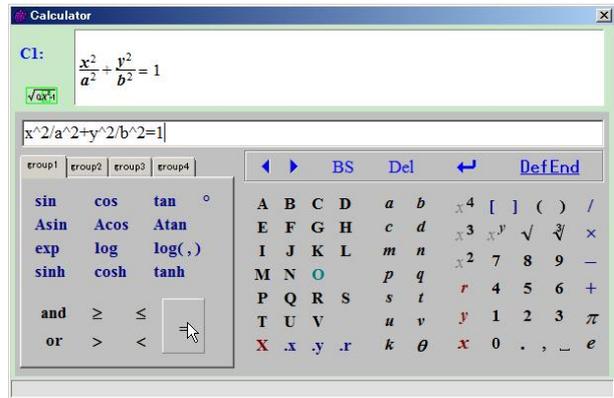
1. Click on [Draw] in the Relation Area.



2. Enter the equation with the Scientific Calculator.

Enter the equality equation from the keyboard or with the buttons on the scientific calculator.

When input is complete, click on [DefEnd].

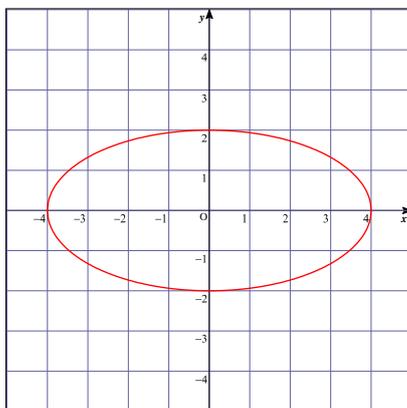
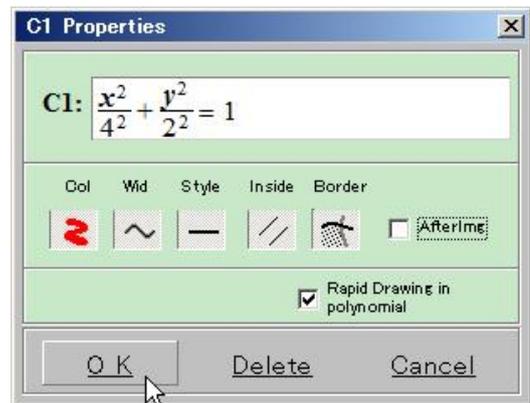


3. Set the graph style

The Relation Property Window is displayed.

See '2-2 Function Graph Properties' for methods of changing the graph color and thickness.

When all the settings have been entered, click on [OK].

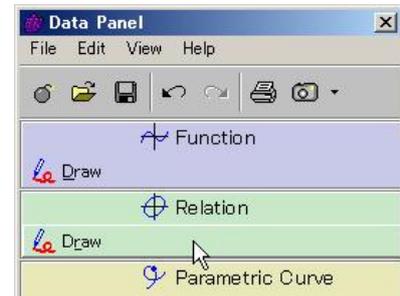


Up to nine relations may be drawn.

4-2 Inequality Area

- Inequality Area

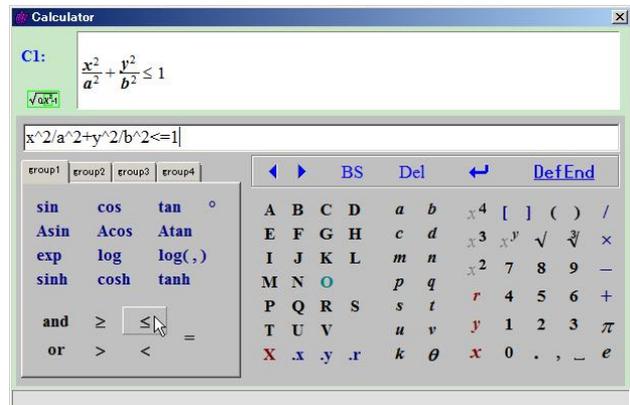
1. Click on [Draw] in the Relation Area.



2. Enter the equation with the Scientific Calculator.

Enter the inequality equation from the keyboard or with the buttons on the Scientific Calculator.

When input is complete, click on [DefEnd].

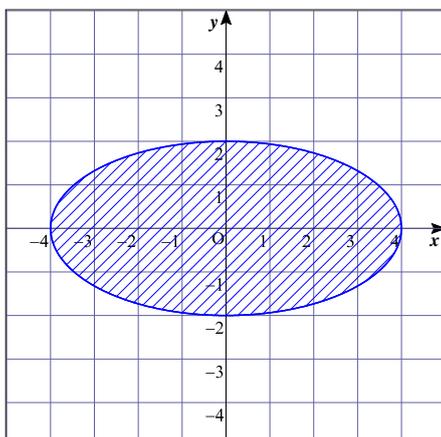
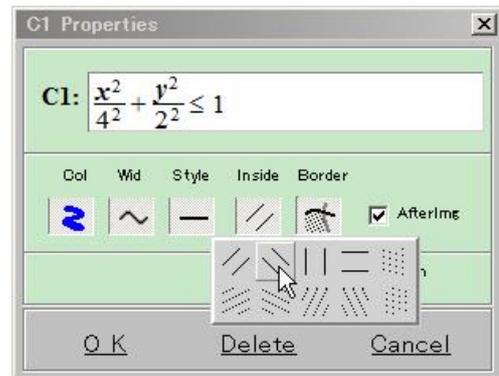


3. Properties of the graph style

The Relation Property Window is displayed.

The hatching pattern for the area may be selected in addition to the graph color and thickness.

When all the settings have been entered, click on [OK].



4-3 Union of Overlaps of Multiple Areas

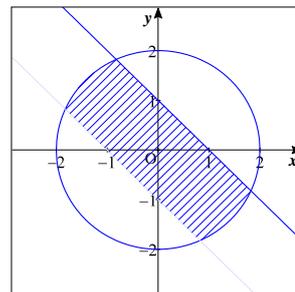
- Displaying Overlap of Multiple Areas

Joins the inequality equation for each area with AND.

Example : $(x^2 + y^2 \leq 2^2)$ AND $(-1 < x + y \leq 1)$

The above may be expressed as C1 : $x^2 + y^2 \leq 2^2$, C2 : $-1 < x + y \leq 1$

C3 : C1 AND C2



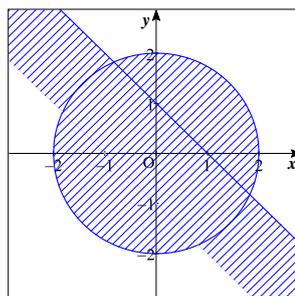
- Displaying Unions of Multiple Areas

Joins the inequality equation for each area with OR.

Example : $(x^2 + y^2 \leq 2^2)$ OR $(-1 < x + y \leq 1)$

The above may be expressed as C1 : $x^2 + y^2 \leq 2^2$, C2 : $-1 < x + y \leq 1$

C3 : C1 OR C2



☆ AND takes priority when AND and OR are mixed.

☆ Parentheses cannot be used for nesting of union and common sets.

The following expressions are therefore not permitted.

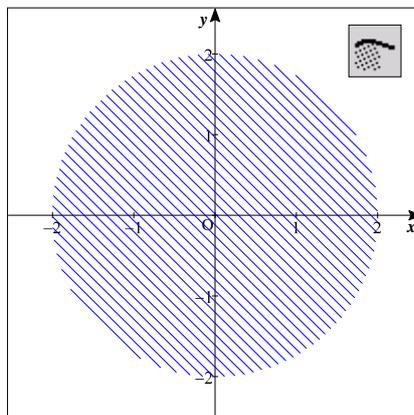
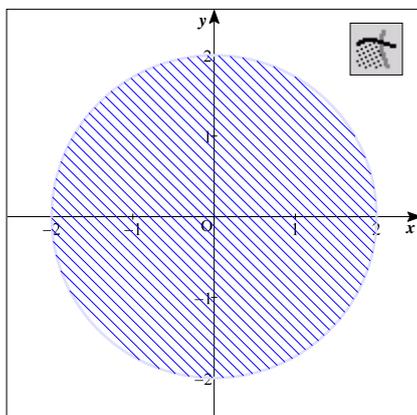
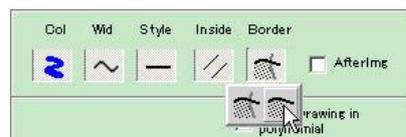
Example 1 : $(C1 \text{ OR } C2) \text{ AND } C3$

Example 2 : $C1 \text{ OR } (C2 \text{ AND } C3)$

Note that the area in Example 2 can be drawn as $[C1 \text{ OR } C2 \text{ AND } C3]$.

- Handling of Boundary Lines

As with $x^2 + y^2 < 2^2$, when the boundary line expressing the inequality equation is not included in the area, the boundary line is displayed in a pale colour. Manipulation of function properties allows display of the pale boundary lines to be suppressed.



4-4 How to Draw Relations

- $y = f(x)$ Type Functions

These are the actual Relations, and are therefore the easiest to draw.

- Conic Curves

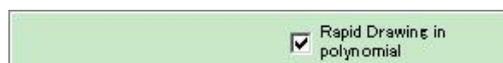
Conic curves are obtained by solving quadratic equations and converting to explicit equations.

For functions other than the two types noted above, find the function value at each point on the screen, and draw after investigating the boundary line between positive and negative areas.

- 10-dimensional and Less Integer Functions

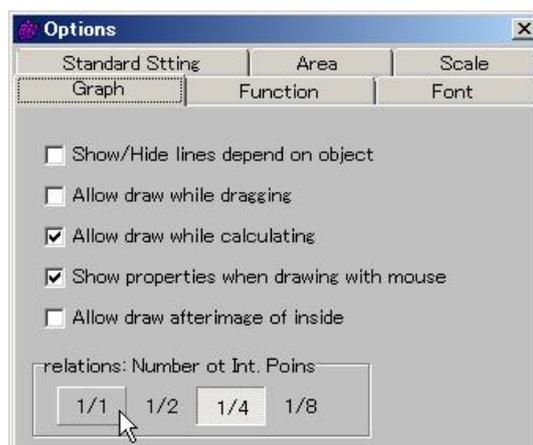
Lagrange interpolation is used for calculation when x or y is a polynomial expression of ten dimensions or less. Display is a simple matter.

☆ With complex functions it becomes difficult to evaluate whether or not they are polynomial expressions of ten dimensions or less, and it may not be possible to draw accurate graphs. In this case, remove the check from [Rapid Drawing in polynomial] in the Relation Properties Window.



- General Functions

Some functions require considerable time for calculation. The density of points on the screen is therefore reduced. If an accurate graph cannot be obtained, try changing [relations: Number of Int.Points Relation] in the graph settings.



Drawing of relations assumes that functions are continuous. For functions including fractions, rearrange the fraction to obtain a continuous function by multiplying a denominator(s) for both side.

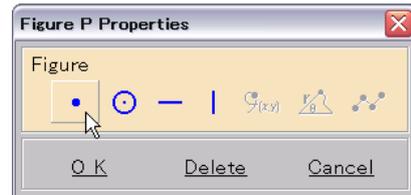
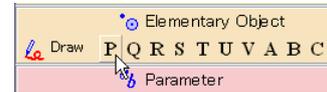
Example : $x = \tan y \rightarrow x \cos y = \sin y$

Chapter 5 Points and Locus

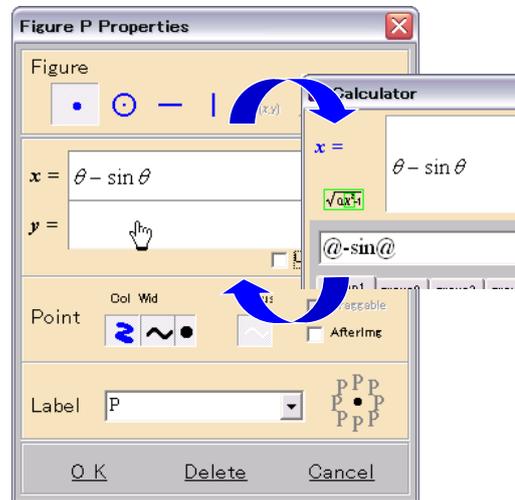
5-1 Drawing a Point

- Drawing a Point

1. Point the cursor to [Draw] in the Graphic Element Area.
2. Select the element name.
3. Select a point from the element types.

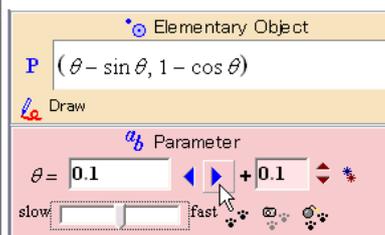
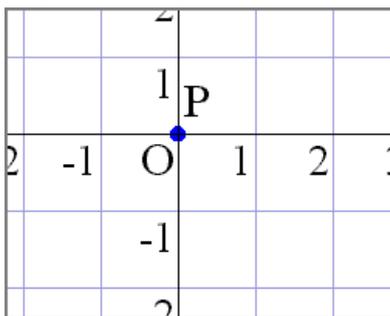


4. Enter an equation for x.
5. Enter an equation for y.
6. Set the graph style.



See the following page for details of changing point color and size.

7. When all the settings have been entered, click on [OK].



Locus cannot be drawn yet. To draw a locus, the afterimage and the thickness of the locus must first be set, and parameters changed.

- Simple Drawing of Points

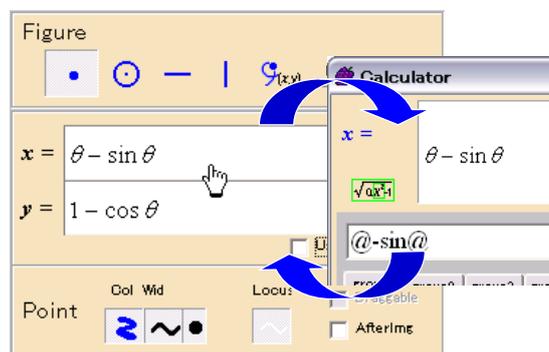
Points can be drawn more simply if coordinates are provided as numerical values rather than as equations,

See '5-7 Points and Dragging'.

5-2 Properties of Points

- Entering and Correcting x Coordinates

1. Click on the x Coordinate Equation Window.
2. Correct the equation with the Scientific Calculator.



- Entering and Correcting y Coordinates

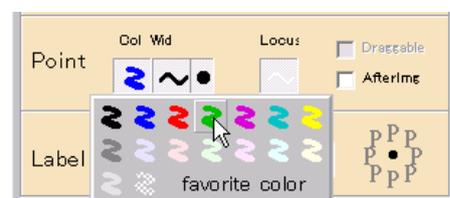
1. Click on the y Coordinate Equation Window.
2. Correct the equation with the Scientific Calculator.

- Changing Point Color and Size

1. Point to the Point Color and Size Window.
2. Select from the palette.

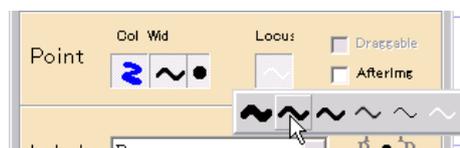
Points may be set as white-on-black by pointing to the right of the Size Window.

See '2-2 Function Graph Properties' for methods of changing the graph color and thickness.



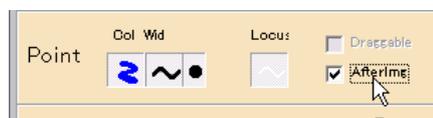
- Changing Locus Thickness

1. Point to the Locus Thickness Window.
2. Select the desired thickness from the palette.



- Setting the Image

Place a check in the [AfterImg] checkbox.



- Setting the Label Display

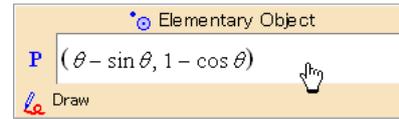
Click at the position to display the label.



5-3 Locus and Afterimages

- Locus and Afterimages

1. Click on the equation display section of the Graphic Element Area.

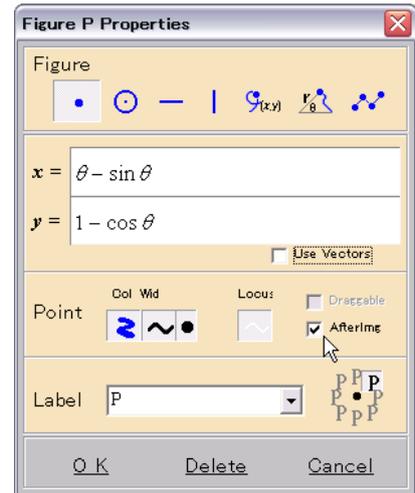


2. Select the locus thickness.

3. Place a check in the [AfterImg] checkbox.

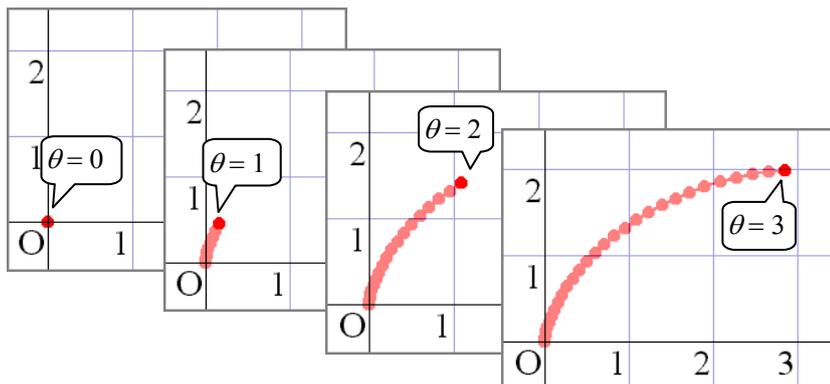
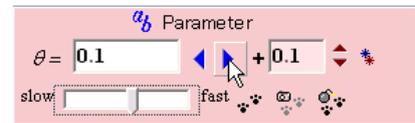
The image may be left even without an image of the point.

4. Click on [OK] to finish.



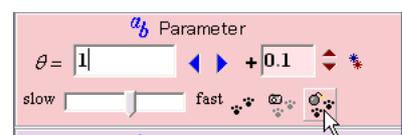
5. Change the parameter.

When the parameter is changed the point image remains and the locus is drawn.



- Deleting the Image and the Locus

Click on the  button (Clear All Afterimage).



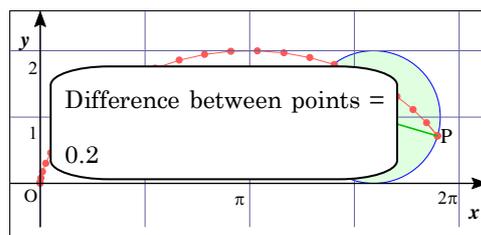
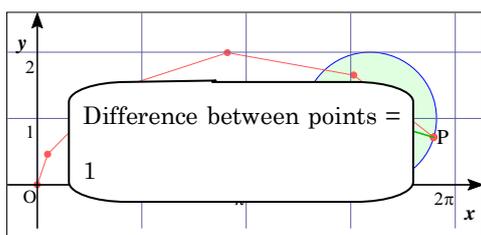
5-4 Making Use of Locus

Loci are broken lines joining points.

Existing loci may only be hidden, deleted, or changed in density.

- Drawing Smooth Locus

Loci simply join points, and are therefore drawn smoothly by reducing the difference between points.



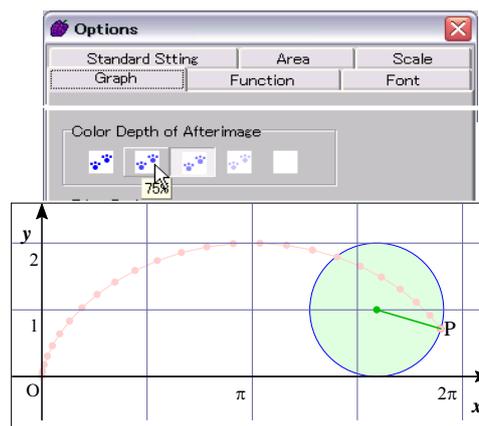
- Changing Locus Density

1. Click on [Options] on the Afterimage Palette.

2. Select image density in the graph settings.

This change is applied to all images and loci.

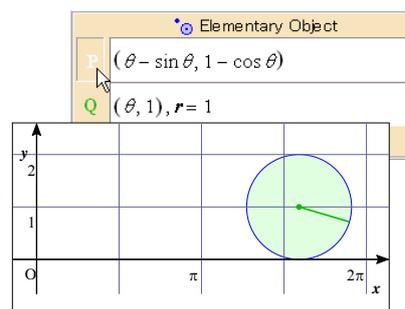
Color and thickness of existing locuses and images cannot be changed.



- Hiding Specific Afterimages and Loci

Click on the element name in the Graphic Element Area.

Element names are assigned buttons in the Graphic Element Area. These are referred to as 'display switches'. Use display switches to permit instantaneous switching between display and non-display of elements.



- Deleting Specific Afterimages and Loci

1. Right-click on the function equation display section of the Data Panel.

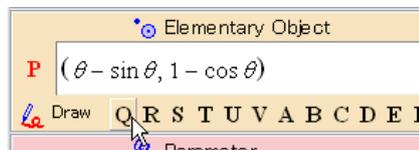
2. Select the afterimage to delete.

5-5 Drawing a Circle

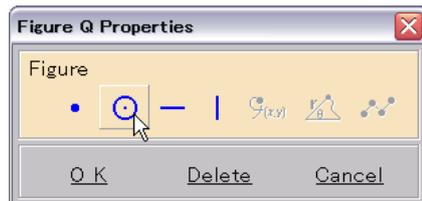
- Drawing a Circle

1. Point to [Draw] in the Graphic Element Area.

2. Select the element name.



3. Select Circle from the element types.



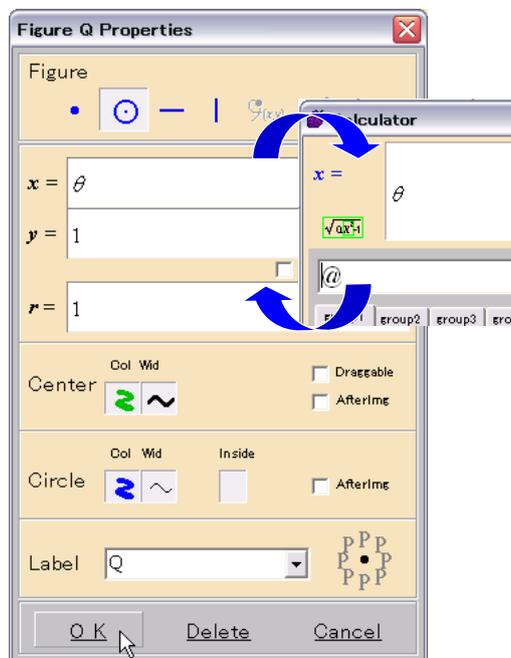
4. Enter an equation for x and y of the circle center.

5. Enter an equation for the radius r.

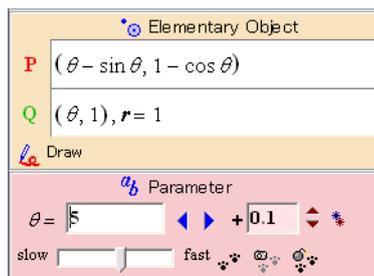
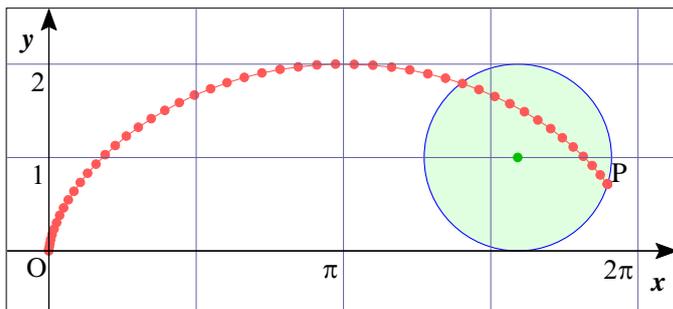
6. Set the color and size of the center.

See '2-2 Function Graph Properties' for methods of changing color and thickness.

7. Set the color, thickness, and inside color of the periphery.



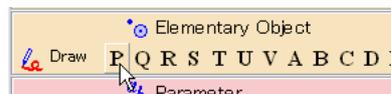
8. Click on [OK].



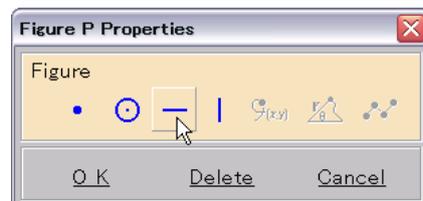
5-6 Drawing Horizontal and Vertical Lines

- Drawing Horizontal Lines

1. Point to [Draw] in the Graphic Element Area.



2. Select the element name.



3. Select a horizontal line from the element types.

4. Enter the y coordinate of the horizontal line.

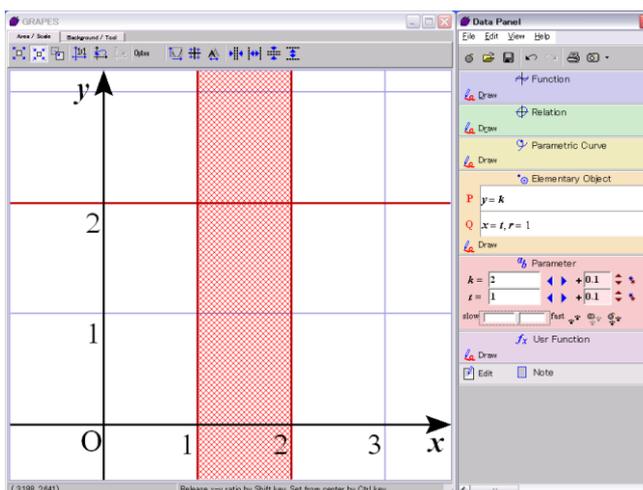
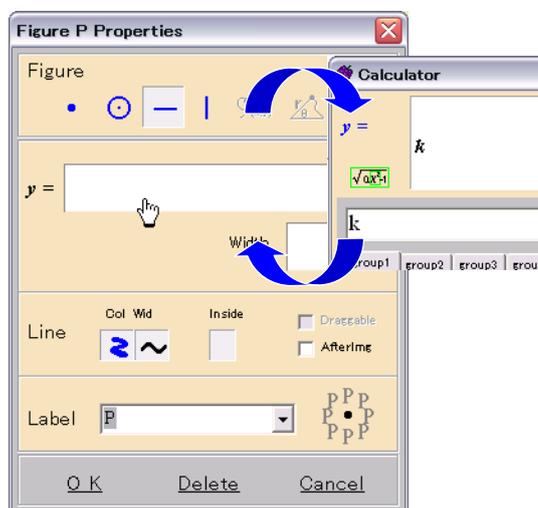
5. Enter the width if required.

6. Set the color and size of the line.

See '2-2 Function Graph Properties' for methods of changing color and thickness.

7. Set the inside color if the line has a width.

8. Click on [OK].



- Drawing Vertical Lines

Same as for horizontal lines.

5-7 Points and Dragging

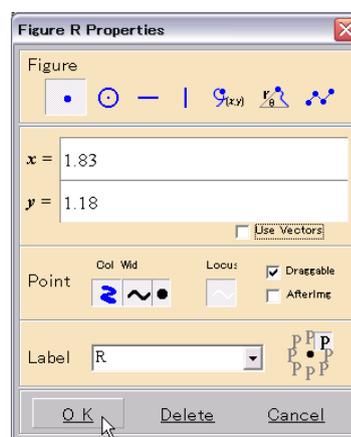
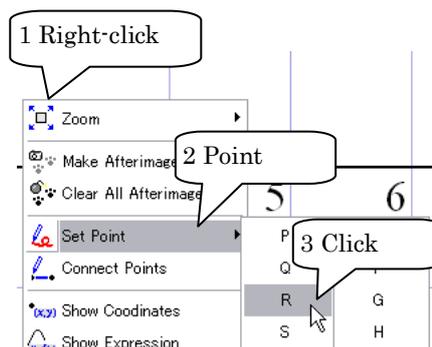
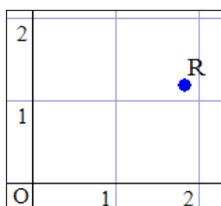
● Drawing a Point

1. Right-click at the position to draw the point.
2. Point to [Set Point].
3. Select the point and click.

The Point Property Window is displayed.

4. Set the color and size of the point.
5. Click on [OK].

Point R has been selected here, however the same applies to all points.



● Dragging a Point

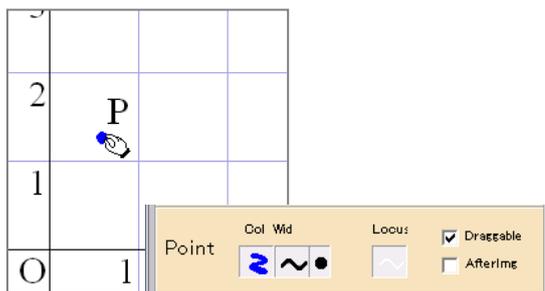
When a numerical value is the basis of a basic graphic element it may be moved by dragging.

1. Point to the point.

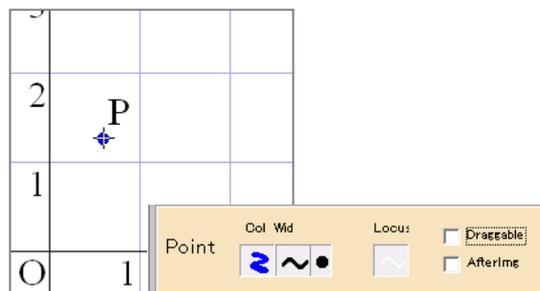
The shape of the cursor changes when the point is able to be dragged (see diagram at bottom-left).

2. Drag with the left button.

- a. The point may be dragged when a check has been placed in the the [Draggable].



- b. The point cannot be dragged when a check has not been placed in the [Draggable] checkbox.



● Dragging a Circle

The circle moves when its center is dragged. The circle radius changes when its periphery is dragged.

● Dragging a Point on a Curve

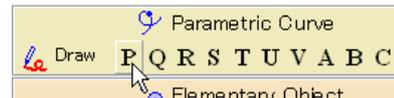
A point on a curve may be dragged. See '6-1 Parameter Display Curves'.

Chapter 6 Curves

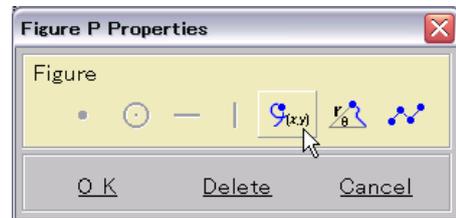
6-1 Parameter Display Curves

- Drawing a Parameter Display Curve

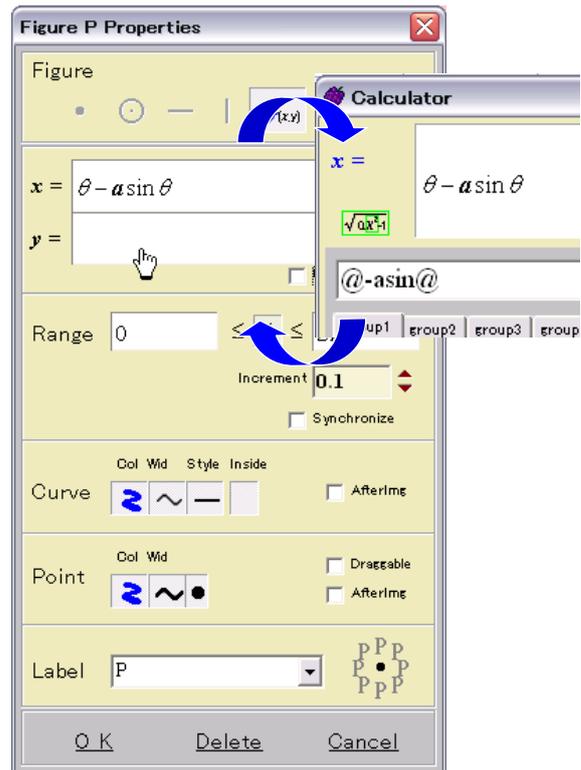
1. Point to [Draw] in the Parametric Curve Area.
2. Select the element name.



3. Click on the  button (Curve) in Element Types.



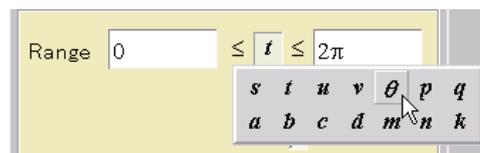
4. Enter a function equation for x.



5. Enter a function equation for y.

6. Set the notation for the parameter.

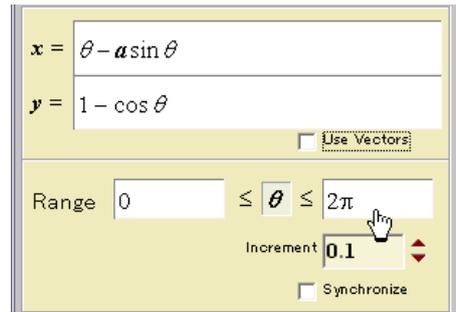
- a. Point to the Parameter Notation Window in the Variable Range Area.



- b. Select the parameter notation from the Notation Pallet.

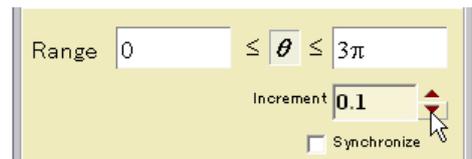
7. Set the variable range for the parameter.

Click on the Variable Range Window and enter the variable in the Scientific Calculator. The top and bottom limits of the variable range may be set individually.



8. Set the range of increase/decrease.

The curve becomes smoother as the range of increase/decrease is reduced. Note that GRAPES requires that (variable range top limit - bottom limit) / range of increase/decrease ≤ 5000 .



★ [If you check “Synchronize”, the value of parameter of Data Panel will be restricted with the range set here. Also note that the increment value set here is synchronized with that of parameter of Data Panel.

9. Set the curve properties.

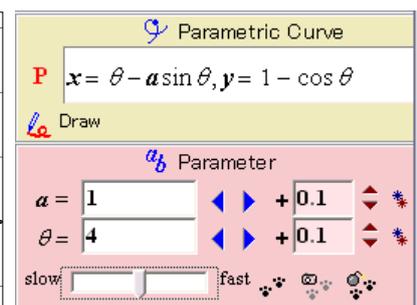
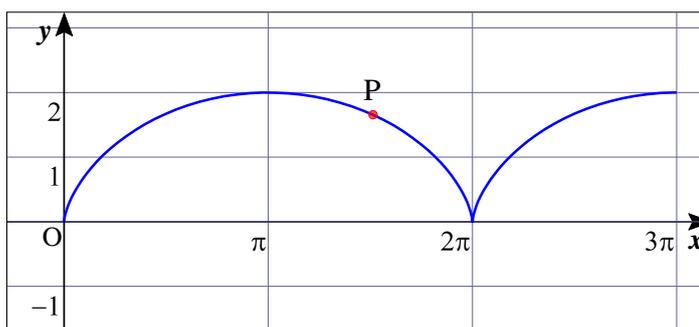
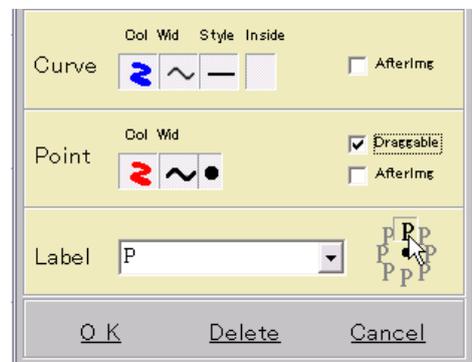
Set color, thickness, line type, inside color, and image.

See ‘2-2 Function Graph Properties’ for methods of changing color and thickness.

10. Set the point properties.

Points moving on a curve may be drawn.

★ Points on the curve may be dragged when [Draggable] is checked.



11. Set the label.

Set the label notation and the display position.

12. Click on [OK].

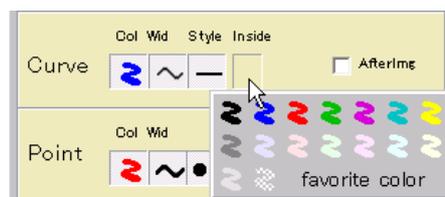
6-2 Painting Inside the Curve

- Inside Painting

The inside of parameter display curves and polar equation graphs may be painted. Note that since the inside of the curve is transparent by default, this setting must be changed before painting is possible.

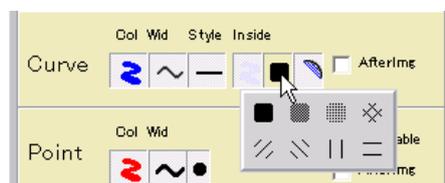
- Changing the Inside Color

1. Point to the Inside Color Window.
2. Select the desired color from the palette.



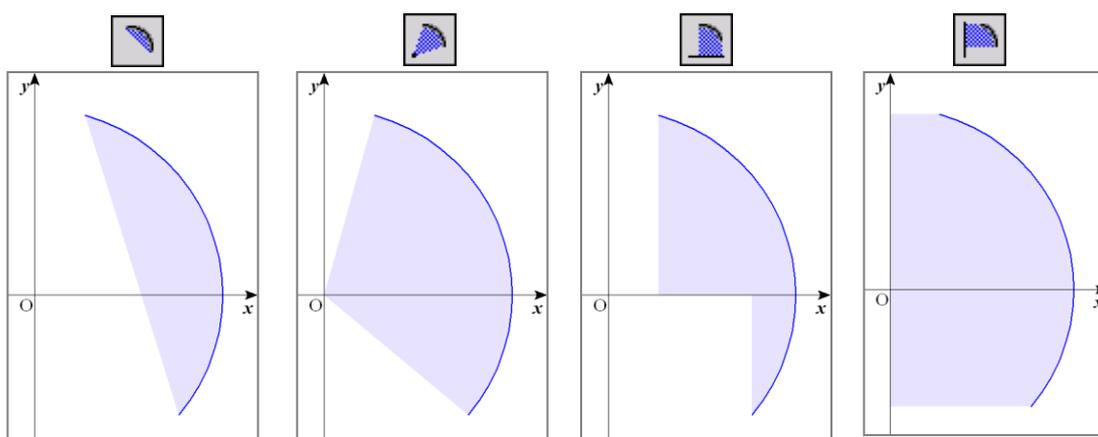
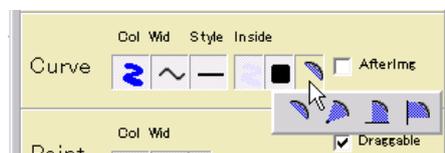
- Changing the Paint Pattern

1. Point to the Pattern Window.
2. Select the pattern from the palette.



- Changing the Paint Area

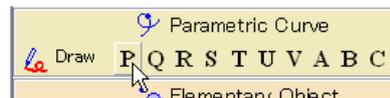
1. Point to the Paint Area Window.
2. Select the desired paint area from the palette.



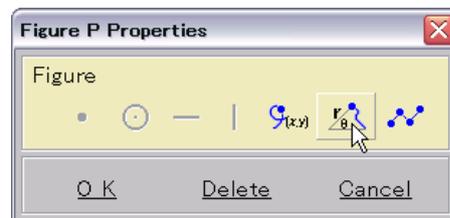
6-3 Polar Equation Graphs

- Drawing a Polar Equation Graph

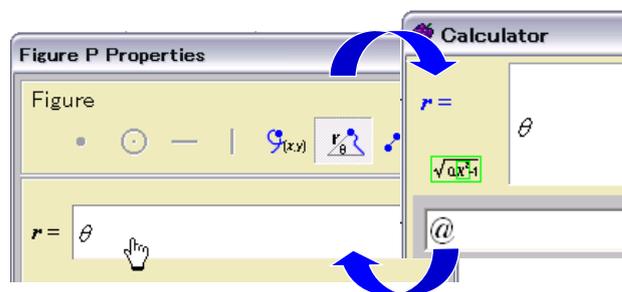
1. Point to [Draw] in the Curve Area.
2. Select the element name.



3. Click on the  button (Polar Equation) in Element Types.

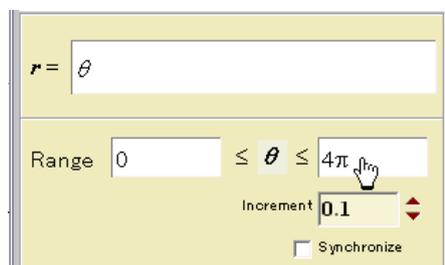


4. Enter an equation for r using θ .



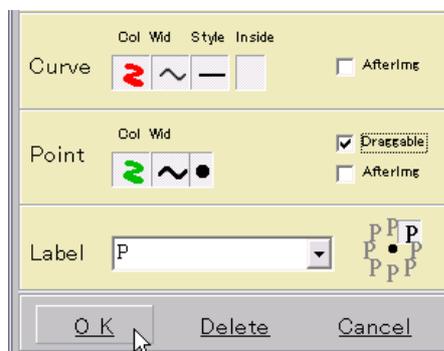
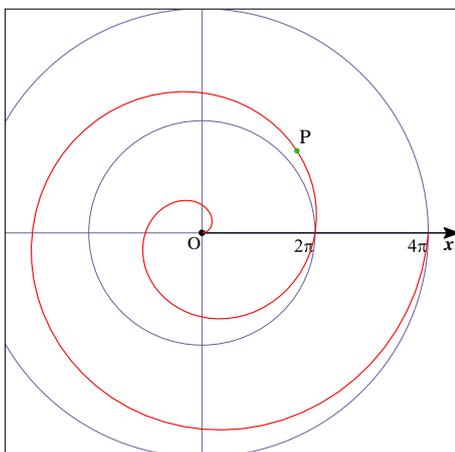
5. Set the variable range and range of increase/decrease for θ .

Click on the Variable Range Window and enter the variable in the Scientific Calculator. The top and bottom limits of the variable range may be set individually.



6. Set the curve, point, and label properties.

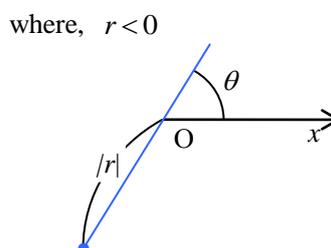
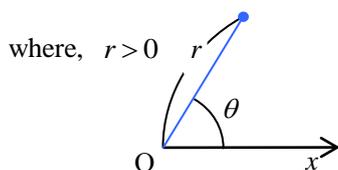
7. Click on [OK].



6-4 Negative Radius Vectors

- What is a Negative Radius Vector?

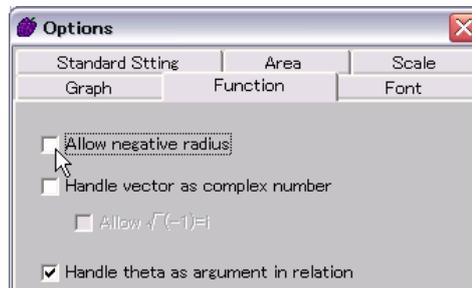
With polar coordinates, the location of a point is expressed in terms of the radius vector – the distance from a pole (the origin), and the angular displacement – the angle from the start line (the x axis). The radius vector can therefore only be a value of 0 or greater. On the other hand, it is sometimes convenient to allow negative radius vectors for polar equations. In such cases, points are taken in the direction opposite to the conventional direction (see diagram at bottom-right), and the radius vector is negative at initialization. $(-r, \theta) = (r, \theta + \pi)$ when a negative radius vector is allowed.



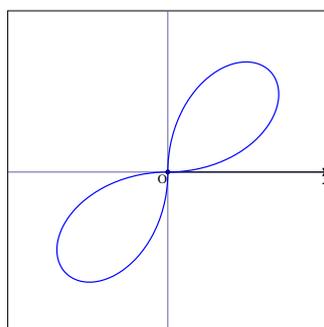
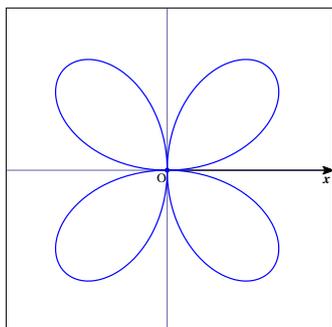
- Disallow Negative Radius Vectors

Remove the check from [Allow negative radius] under the [Function] tab in the Options Window.

The following graphs for $r = \sin 2\theta$ illustrate the cases of a negative radius vector allowed (left), and not allowed (right).



In subsequent graphs, the part for which the radius vector is negative will not be drawn.

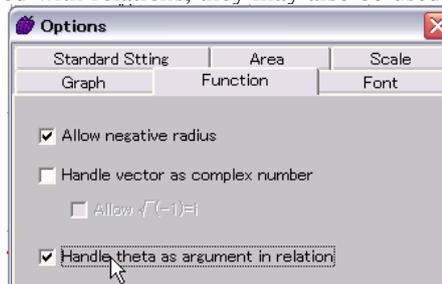


6-5 Relations and Polar Equations

Since the radius vector r and angular displacement θ may be used with relations, they may also be used in drawing graphs of polar equations.

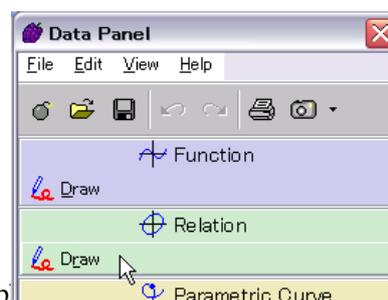
- Relations and Polar Equations

1. Place a check in the [Handle theta as argument in Relation] checkbox under the [Function] tab in the Options Window.



θ is then angular displacement rather than a parameter.

2. Click on [Draw] in the Relation Area.

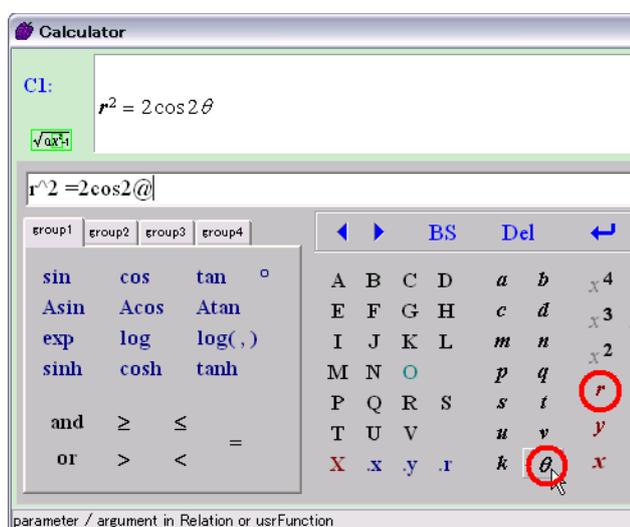
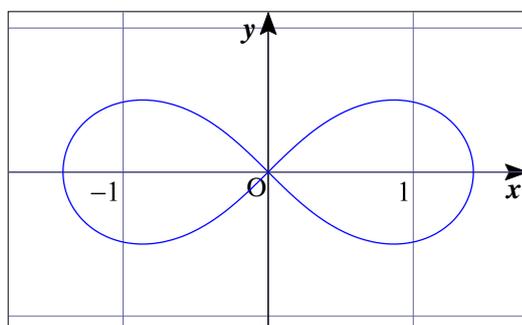


3. Enter the equation with the Scientific Calculator.

Enter the equation using the radius vector r , and the angular displacement

(a certain portion is hidden in the above sentence)

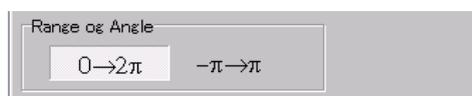
4. Subsequent operation is the same as for Relation graphs.



- Handling the Radius Vector and Angular Displacement in Relations

The range of angular displacement θ is $0 \leq \theta < 2\pi$ or $-\pi \leq \theta < \pi$. In either case, it is set in [Range of Angle] under the [Function] tab in the Options Window.

Negative radius vectors are not allowed in relations.



Chapter 7 Linked Graphic Elements

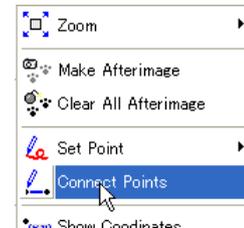
7-1 Linking Two Points

- Linking Two Points

1. Right-click in the Graph Display Area and click on the

 (Connect Points) button on the pop-up menu.

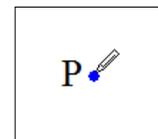
This selects the Link Points Mode.



The Connect Points Mode is cleared by clicking again, or by clicking on the Enlarge, Shrink, or Move buttons.

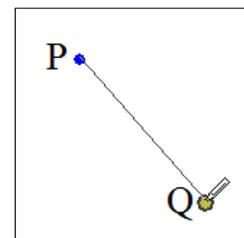
2. Point to one of the points with the mouse.

The cursor changes to a pen icon when the cursor is moved over the point.



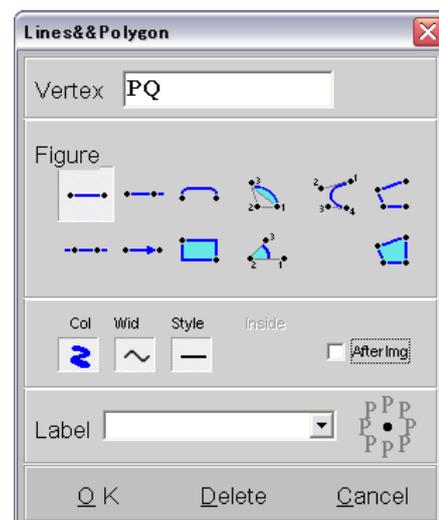
3. Specify the two ends of the line segment.

- Left-click on one end of the line segment.
- Drag while holding the left button down.
- Release at the other end.

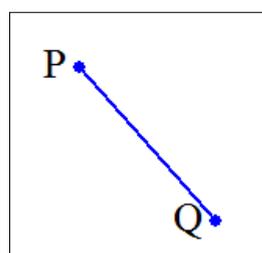


4. Line Segment Settings

Set Graphic Element type, Color, Thickness, Area Color, and Image in the Linked Graphic Element Property Window.



5. Click on [OK] to finish.

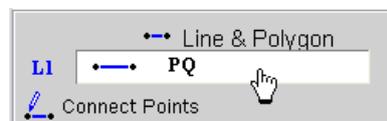


7-2 Changing and Deleting Linked Graphic Element Properties

- Changing and Deleting Linked Graphic Elements

1. Click on the relevant graphic element in the Linked Graphic Element Area.

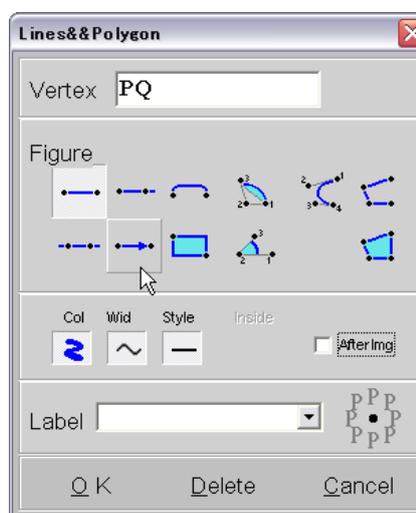
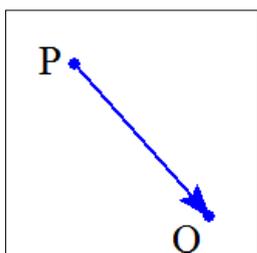
Or, right-click on the linked graphic element and then click on [Linked Graphic Element Properties] on the pop-up menu.



2. Set the type, color, thickness, area color, and image of the graphic element.

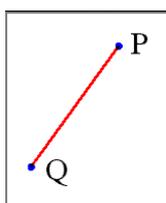
Click on [Delete] to delete.

3. Click on [OK].

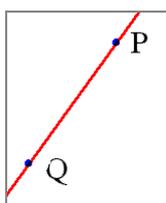


- Graphic Element Types

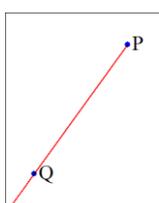
Line segment



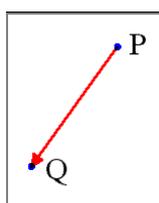
Straight line



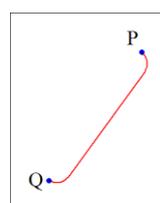
Half-line



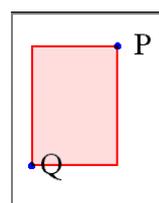
Arrow



Parenthesis



Rectangle



Deletion of a single point in a graphic element will delete the element.

7-3 Drawing Broken Lines and Polygons

- **Displaying Broken Lines and Polygons**

Three or more points must be drawn before starting. In the example below the method of drawing a triangle based on the three existing points P, Q, and R is described.

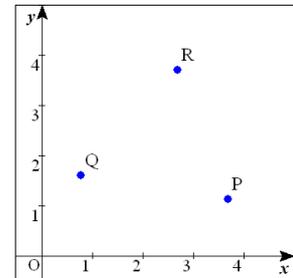
1. Right-click in the Graph Display Area and click on the

 (Connect Points) button on the pop-up menu.

This selects the Link Points Mode.

2. Link the two points P and Q.

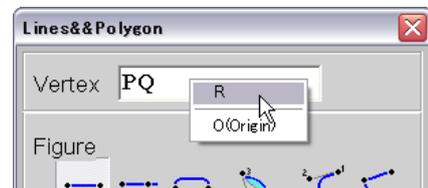
Click on the left button on the point P, drag to Q while holding the button down, and release.



The Linked Graphic Element Property Window is displayed.

3. Click on the Apex Window.

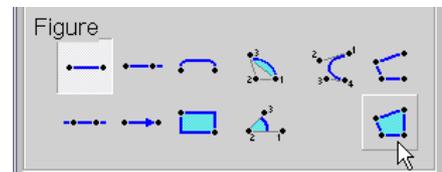
The pop-up window is opened to display a list of apexes which may be specified.



4. Select [R] from the pop-up menu.

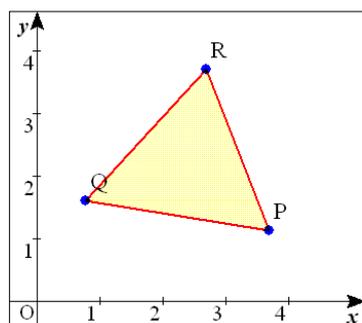
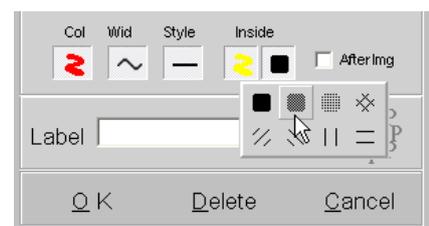
5. Click on the  button (Polygon).

Click on the  button (Broken Line) to draw a broken line.



6. Set the boundary color, inside color, and hatching pattern.

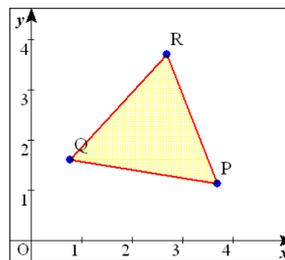
7. Click on [OK].



7-4 Displaying Angles

- Displaying Angles

Three or more points must be drawn before starting. In the example below, the method of drawing $\angle PQR$ based on the existing triangle PQR is described.



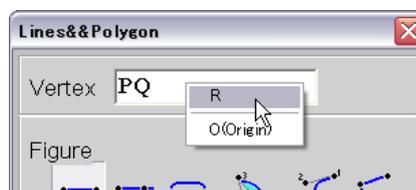
1. Select the Link Points Mode.

See above for details of the Link Points Mode.

2. Link the two points P and Q.

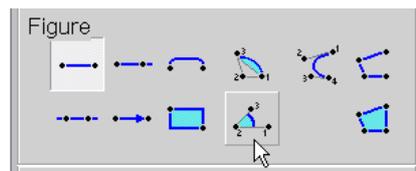
3. Click on the Apex Window.

4. Select [R] from the pop-up menu.

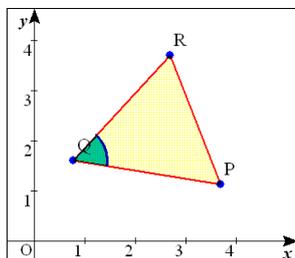
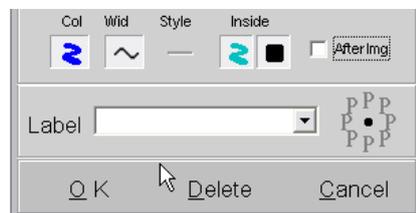


5. Click on the  button (Angle).

6. Set the line color and inside color.



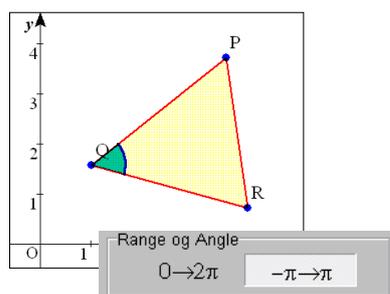
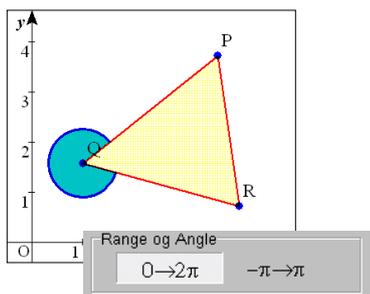
7. Click on [OK].



- Orientation of Angles

Angles are displayed in counter-clockwise orientation. For example, $\angle PQR$ is formed beginning with the half-line QP, proceeding counter-clockwise to the half-line QR.

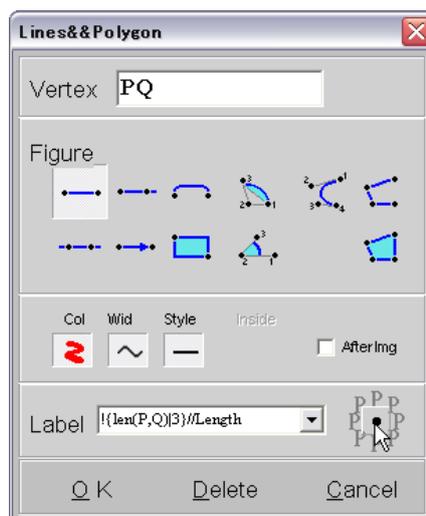
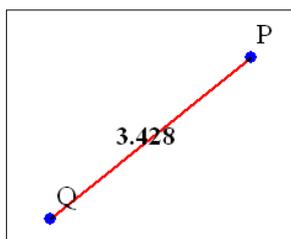
To display only angles of less than 180° , set [Range of Angle] under the [Function] tab in the Options Window to $[-\pi \rightarrow \pi]$.



7-5 Displaying Line Segment Length and Angle Size

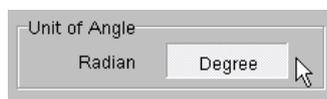
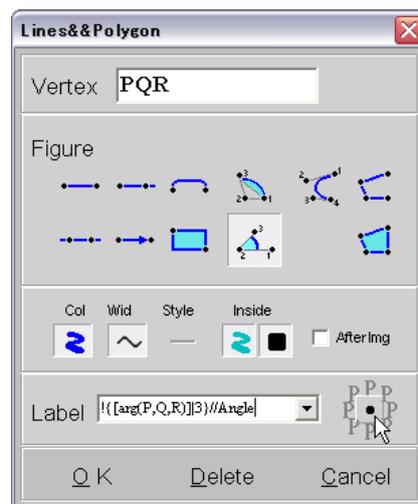
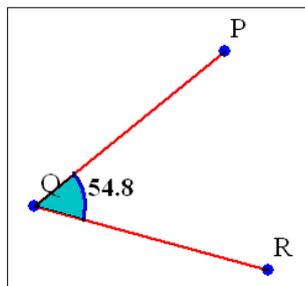
- Displaying Line Segment Length

1. Click on the line segment PQ.
2. Click on [Linked Graphic Element Properties].
3. Select “ $!{\{[PQ]\}}$ ” from the pull-down menu in the Label Window.
4. Click on the display position of the label.



- Displaying Angle Size

1. Right-click on the arc of the angle.
2. Click on [Linked Graphic Element Properties].
3. Select “ $!{\{[arg(P,Q,R)]\}}$ ” from the pull-down menu in the Label Window.
4. Click on the display position of the label.
5. Set [Unit of Angle] under the [Function] tab in the Options Window to [Degree].



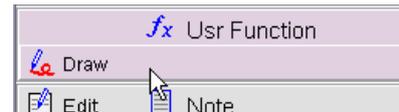
Chapter 8 Mutual Referencing of Function Graphs

8-1 Using Defined Functions

A function may be defined as $f(x)$ when it is to be used repeatedly. Up to five of these defined functions may be used.

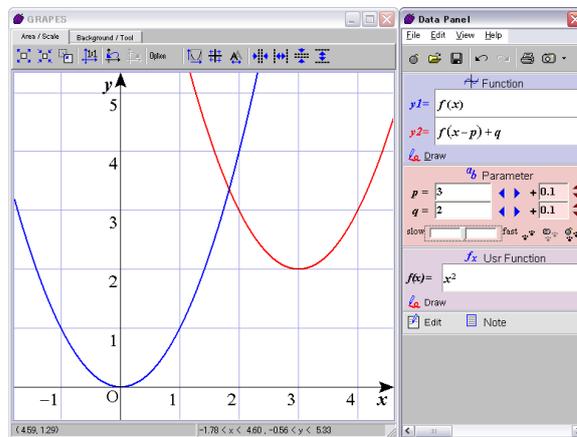
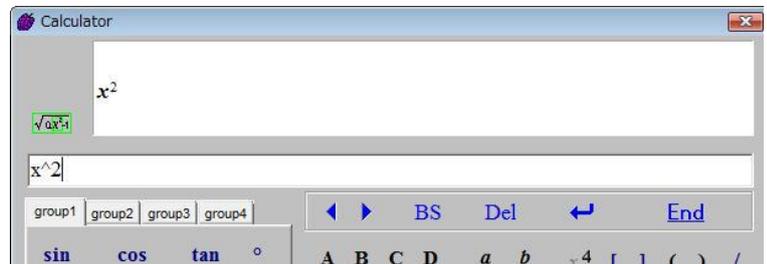
- Defined Functions

1. Click on [Draw] in the Defined Function Area.

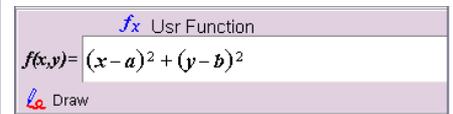


2. Enter the equation with the Scientific Calculator.

Eight defined functions ($f, g, h, f1, \dots, f5$) are available.

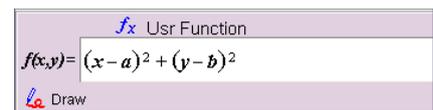


A graph of the two functions $y = f(x)$ and $y = f(x - p) + q$



- Bivariate Functions

A function including the variable y is handled as a bivariate function. In this case, the function label is displayed in the format “ $f(x,y)$ ” to indicate a bivariate function.



Two arguments are passed in the format “ $f(2x,y)$ ” when a bivariate function is referenced. The second argument may be omitted, and in this case it is handled as $y=0$, and therefore $f(x) = f(x, 0)$.

Up to four arguments (x, y, z, w) may be used, and the function is therefore $f(x, y, z, w)$.

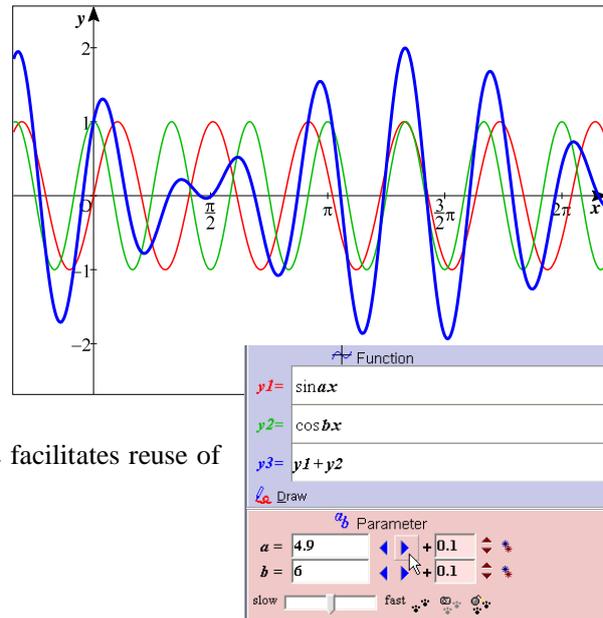
8-2 Referencing Functions

- Referencing Functions

The diagram at left shows the graphs of two functions $y1 = \sin ax$ and $y2 = \cos bx$, and the synthesized function $y3 = \sin ax + \cos bx$.

In this case, processing speed is increased if the format: $y3 = y1 + y2$ is used.

Furthermore, when investigating synthesis of another function, simply substituting for $y1$ and $y2$ facilitates reuse of data.



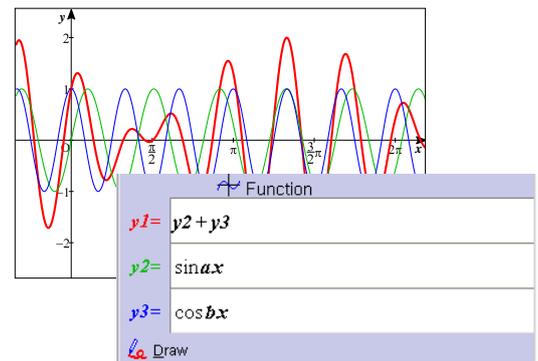
- Reference Sequence

Care must be taken with the sequence in which the function is referenced.

For example, the same graph may be drawn with

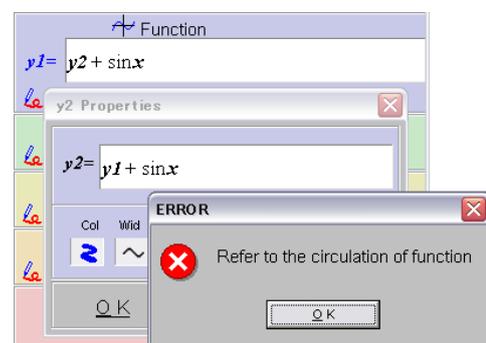
$$y1 = y2 + y3$$

$$\text{and } y2 = \sin ax, \quad y3 = \cos bx$$



- Circular Referencing

The type of referencing able to be used is restricted. Care is required with circular referencing of functions. An error message is displayed when circular referencing is found, and input is cancelled.



Functions may be referenced from relations, however processing speed is reduced. Functions cannot be referenced from basic graphic elements and curves.

8-3 Dragging Graphs – Coordinate Referencing

Point coordinates may be referenced from a function.

For example, a quadratic equation moved by dragging the apex A may be drawn.

- Dragging a Graph

1. Draw point A

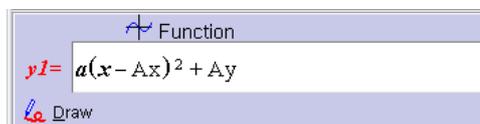
See '5-7 Points and Dragging' for details of drawing points.

2. Use the point coordinates to draw the graph of the quadratic equation.

The coordinates of the point A are (A.x, A.y).

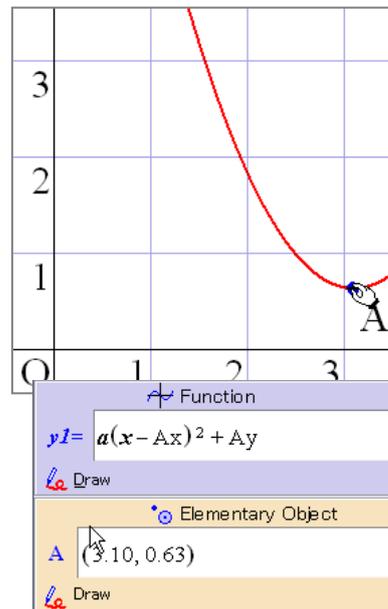
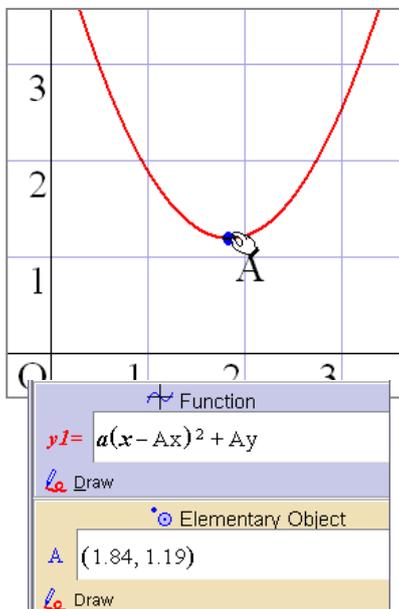
The quadratic equation with the apex at point A is therefore:

$$y = a(x - A.x)^2 + A.y .$$



3. Drag point A.

The graph moves when the mouse button is released.



8-4 Linking Graphic Elements

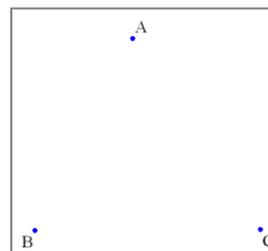
Point coordinates may be referenced from another function. For example, draw a triangle and its center of gravity.

Moving the apex also moves the center of gravity.

- Referencing the Components of a Basic Graphic Element

1. Draw the apexes A, B, and C.

Draw the three points A, B, and C.



2. Link the apexes A, B, and C.

Edges are unnecessary, however drawing them gives the triangle form.

See '7-3 Drawing Broken Lines and Polygons' for details of drawing triangles.

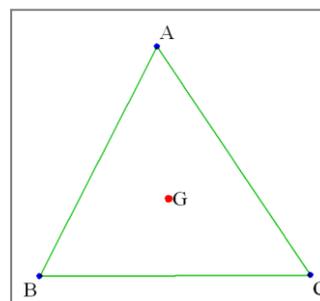
3. Determine the center of gravity G.

The coordinates of points A, B, and C are

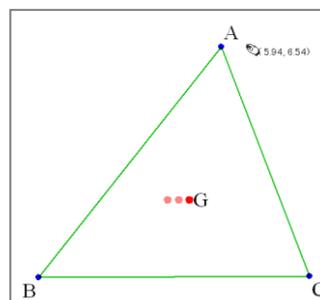
$$(A.x, A.y), (B.x, B.y), (C.x, C.y)$$

and the coordinates of the center of gravity G are therefore

$$\left(\frac{A.x + B.x + C.x}{3}, \frac{A.y + B.y + C.y}{3} \right)$$



● Elementary Object	
A	(1.13, 2.22)
B	(-1.05, 0.35)
C	(1.58, -0.68)
G	$\left(\frac{Ax+Bx+Cx}{3}, \frac{Ay+By+Cy}{3} \right)$
Draw	



4. Move the apex.

The center of gravity G also moves.

The center of gravity of the triangle A, B, and C may be expressed concisely with the use of vectors. This is described in 'Chapter 9 Using Vectors'.

Chapter 9 Using Vectors

9-1 Points and Vectors

- Points and Vectors

The center of gravity G of the triangle ABC is expressed in coordinates as follows.

$$\left(\frac{A.x + B.x + C.x}{3}, \frac{A.y + B.y + C.y}{3} \right)$$

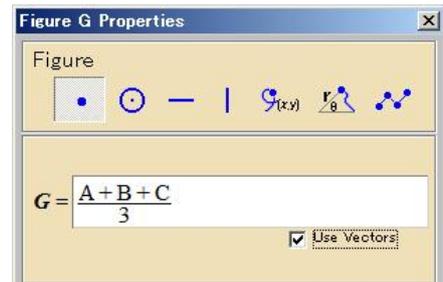
This is expressed in position vectors as follows.

$$\frac{\vec{OA} + \vec{OB} + \vec{OC}}{3}$$

This is also possible in GRAPES.

$$\frac{\vec{OA} + \vec{OB} + \vec{OC}}{3} \text{ or } \frac{A + B + C}{3}$$

Points and vectors are considered to be the same in GRAPES.



- Vectors and Real Numbers

A vector is created from two real numbers enclosed in parenthesis. For example, the vector having an x component of 2 and a y component of 3 is expressed as follows.

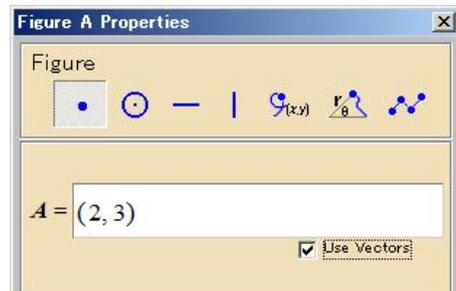
$$(2, 3)$$

This is the same as coordinate notation.

On the other hand, when a vector is provided, 'x' and 'y' are added to the components.

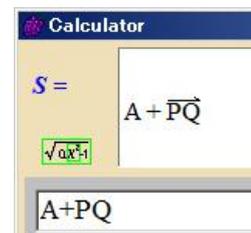
$$(2, 3).x = 2$$

$$(2, 3).y = 3$$



- Points and Displacement Vectors

A displacement vector is expressed by drawing a basic graphic element and points on a curve together. For example, the two points P and Q are expressed as \vec{PQ} .



9-2 Entering in Vector Notation

The use of vectors allows the position of a point to be entered without the need for separate x and y coordinates.

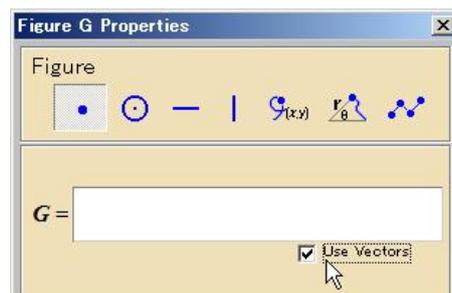
- Entering the Position of a Point Using Vectors

First, draw the triangle ABC before beginning the following operation.

1. Create the new basic graphic element G, and select [Point].

2. Place a check in the [Use Vectors] checkbox.

The two Equation Windows are merged into one.

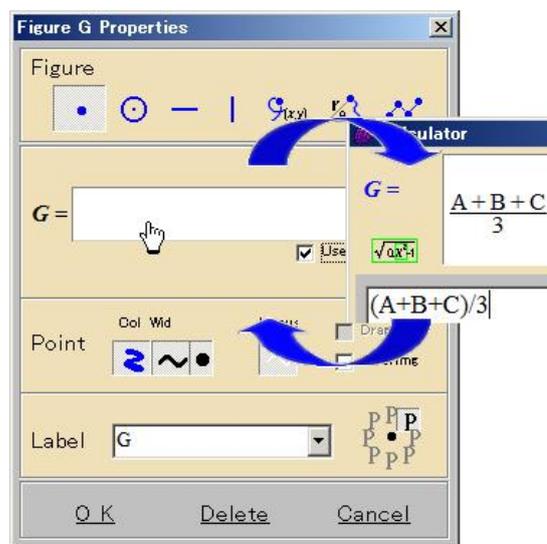


3. Click on the Equation Window.

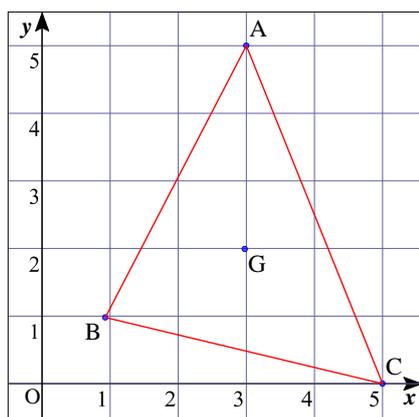
4. Enter the equation with the Scientific Calculator.

The Scientific Calculator is able to handle vector equations. The center of gravity of the triangle ABC is entered in the example at right.

Entering equations using vectors is described later.



5. Click on [OK].



Elementary Object	
A	(3, 5)
B	(0.93, 0.98)
C	(5, 0)
G	$\frac{A+B+C}{3}$

9-3 Vector Calculations

- Vector Addition and Subtraction, and Real Number Multiples

Mathematical notation may also be readily used for vector addition and subtraction, and real number multiples.

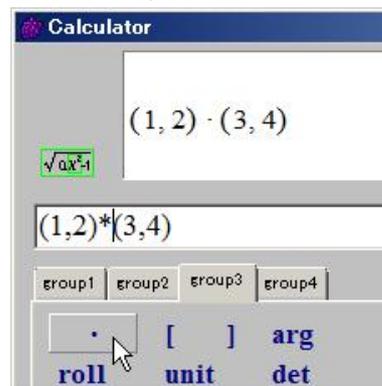
Examples	$P+(-4,5)$:	Sum of the position vector of point P and the vector (-4,5).
	$(2,3) - (-4,5)$:	Difference between the vector (2,3) and the vector (-4,5).
	$3\overrightarrow{PQ}$:	Three times the displacement vector PQ.
	$(A + B)/2$:	Mid-point between points A and B (position vector).

- Dot Products of Vectors

The operator \cdot is used in finding the dot product of two vectors.

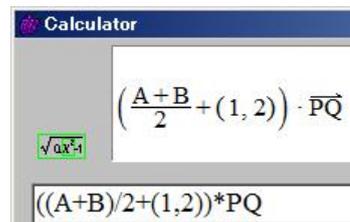
Enter the * symbol on the keyboard.

Example $(1,2) \cdot (3,4)$: Dot product of (1,2) and (3,4).



- Mixed Operation

Complex calculations are possible with the use of parentheses.

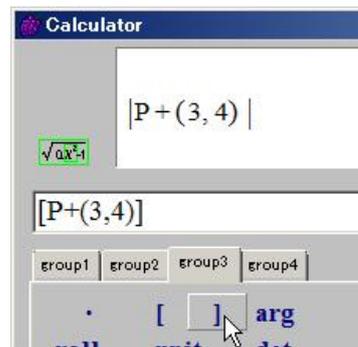


- Size of Vectors

The same symbol as the absolute-value symbol is used in finding vector size (length). This is also mathematical notation.

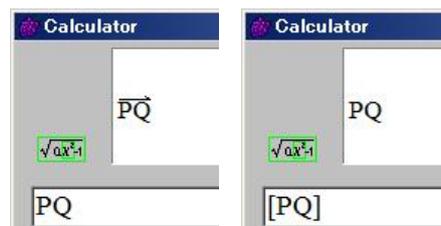
Enter the [and] symbols on the keyboard.

Example $|(3,4)|$: Size of vector (3,4).



- Line Segment Length

'[PQ]' represents the size of the displacement vector \overrightarrow{PQ} , however it is displayed simply as 'PQ'. This represents the length of the line segment PQ.



9-4 Distances and Angles

- The len Function Returns Distance

$\text{len} (P , Q)$: Distance between points P and Q.

$\text{len} (P)$: Distance from origin of point P.

The following coordinates may also be used.

$\text{len} (P , (2 , 3))$: Distance between point P and point (2 , 3).

$\text{len} (2 , 3)$: Distance from origin of point (2 , 3).

Coordinates may also be used in a similar manner with other functions.

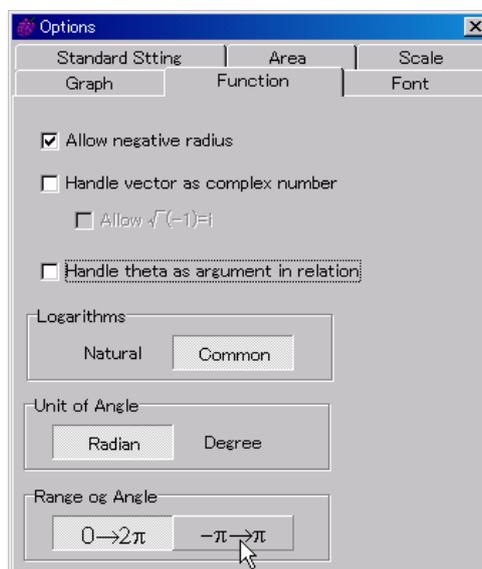
- The arg Function Returns an Angle

$\text{arg} (P , Q , R)$: The size of $\angle PQR$ counter-clockwise.

$\text{arg} (P , Q)$: $\angle POQ$

$\text{arg} (P)$: Angular displacement of OP.

Angles are in units in accordance with the circular method when initialized, and with a range of 0 to 2π . Angle units and range may be changed under the [Function] tab in the Options Window.



- Determinant Values

$\text{det} (P , Q)$: Returns a determinant value created by two vectors.

Function values are real numbers.

9-5 Functions Returning Position

- The mid Function Returns the Mid-point of a Line Segment

$\text{mid}(P, Q, m, n)$: The point dividing the line segment PQ into $m : n$.

$\text{mid}(P, Q)$: The mid-point function 'mid' for the line segment PQ returns the inside and outside mid-points.

Enter 'mid' on the keyboard.

- The intr Function Returns the Intersection Point of Straight Lines

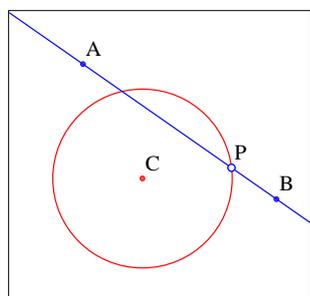
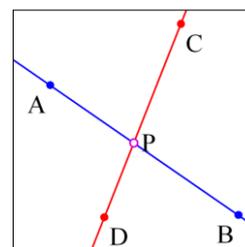
$\text{intr}(A, B, C, D)$: Intersection of the two straight lines AB and CD.

$\text{intr}(A, B, C, r)$: Intersection of the straight line AB and the radius r from the center C.

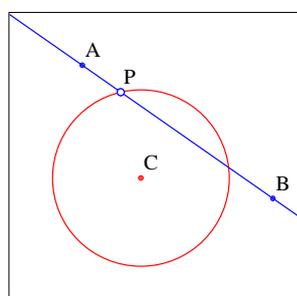
The intr function returns the intersection point of two straight lines.

Enter 'intr' on the keyboard.

☆ Two intersections exist for a straight line and a circle. Use $\text{intr}(B, A, C, r)$ to obtain the other intersection in this case.



$\text{intr}(A, B, C, r)$



$\text{intr}(B, A, C, r)$

$\text{intr}(A, a, B, b)$: Intersection of the radius a from the center A and the radius b from the center B.

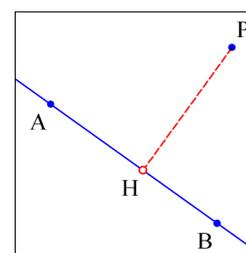
☆ Two intersections exist for two circles. Use (B, b, A, a) to obtain the other intersection in this case.

- The perp Function Returns the Base of a Perpendicular Line

$\text{perp}(P, A, B)$: Base of the perpendicular line from the point P to the straight line AB.

The perp function returns the base of the perpendicular line from a point to a straight line.

Enter 'perp' on the keyboard.



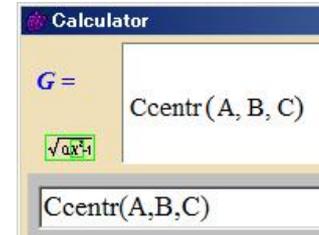
- Functions Returning the Center of Gravity, Circumcenter, Orthocenter, and Inner Center of a Triangle

Gcentr (A, B, C): Center of gravity of triangle ABC.

The Gcentr function returns the center of gravity of a triangle.

Enter 'ccentr' on the keyboard.

Functions returning the circumcenter, inner center, and orthocenter are also available.



Operation	Function name	Key input	Example	Value
Return center of gravity	Center of gravity	Gcentr	Gcentr (A, B, C)	Vector
Return circumcenter	Circumcenter	Ccentr	Ccentr (A, B, C)	Vector
Return orthocenter	Orthocenter	Hcentr	Hcentr (A, B, C)	Vector
Return inner center	Inner center	Icentr	Icentr (A, B, C)	Vector
Return radius of circumcircle	Circumcircle	Crad	Crad (A, B, C)	Real number
Return radius of incircle	Incircle	Irad	Irad (A, B, C)	Real number

- The rot Function Returns Rotation

rot (A, C, t): Rotate point A around point C by angle t.

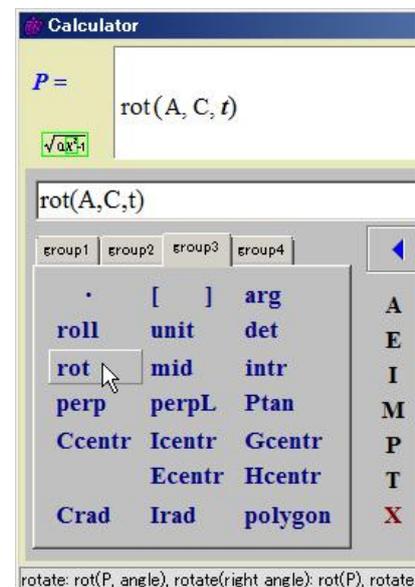
rot (A, t): Rotate point A around the origin by angle t.

rot (A): Rotate point A around the origin by 90°.

The rot function employs vector values (i.e. points).

Enter 'rot' on the keyboard.

The angle of rotation is in units in accordance with the circular method when initialized. Change to the frequency method under a [Function] tab in the Options Window.



- Points on Unit Circles

roll(t): Returns a point $(\cos t, \sin t)$ on a unit circle.

Function values are vectors.

Points expressed in polar coordinates are expressed as a roll(θ) when the roll function is used.

- Unit Vectors

unit(P): Unit vector in the \overrightarrow{OP} direction.

- Points on Edges of Polygons

polygon(t, P_1, P_2, \dots, P_n): Points on edges of a polygon of n angles. Through each angle of the polygon in turn with $0 \leq t < n$ ($n \leq 20$).

When k is an integer, $\text{polygon}(k-1, P_1, P_2, \dots, P_n) = P_k$.

When k is a non-integer it represents a point on an edge.

When $t = t_0 + kn$ (k is an integer) ;

$$\text{polygon}(t, P_1, \dots, P_n) = \text{polygon}(t_0, P_1, \dots, P_n)$$

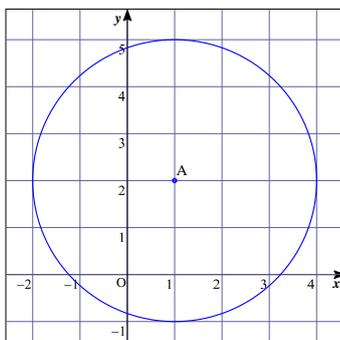
And in particular;

$$\text{polygon}(n, P_1, \dots, P_n) = \text{polygon}(0, P_1, \dots, P_n)$$

- X Represents the Point (x,y)

Variables x and y may be used with relations and defined functions. Point X is a point (x,y) representing the set of these variables.

For example, $XA=3$ in an relation represents a circle with center A and radius 3.



⊕ Relation	
C1:	$XA = 3$
	Draw
⊙ Elementary Object	
A	$(1, 2)$
	Draw

9-6 Defined Functions and Vectors

- Handling Vectors (points) as Arguments

For example, assume that $f(x, y) = 2x + 3y$.

In this case, $f(4,5) = 2 \times 4 + 3 \times 5$, and in the case of $P = (4,5)$, $f(P) = 2 \times 4 + 3 \times 5$.

- Vector X Represents (x,y)

For example, in the $f(x, y) = 2x + 3y$, $2x + 3y = (2,3) \cdot (x, y)$, and the function may therefore be expressed as $f(x, y) = (2, 3) \cdot X$.

Similarly, vector Y is expressed as (z,w).

- The Vector is Returned as a Function Value

Defined function values may be vectors.

For example, if $f(x) = xA + (1-x)B$, the function $f(x)$ returns the inside mid-point of the line segment AB.

- Transforming Vectors (points)

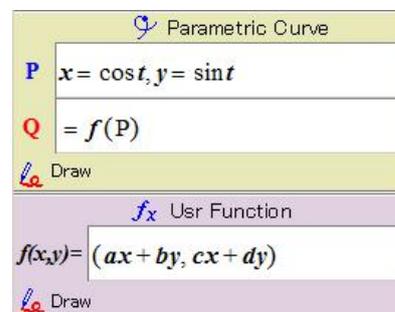
Transformation of points on a flat plane may be expressed in a single function with the use of defined functions.

For example,

$f(x, y) = (ax + by, cx + dy)$ is a one-dimensional transformation.

Since $f(x, y) = f((x, y))$ in GRAPES defined functions, if $Q = f(P)$, the point Q expresses the image of point P.

Similarly, $f(x, y, z, w) = f((x, y), (z, w))$.



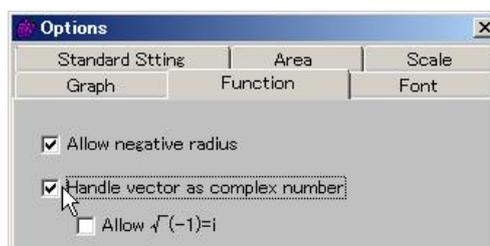
9-7 Vectors and Complex Numbers

Vector data may be handled as complex numbers. In this case, calculations between vectors, and calculations with real numbers, are expanded.

- Complex Number Mode

Place a check in the [Handle vector as Complex number] checkbox under the [Function] tab in the Options Window.

Selecting the Complex Numbers Mode allow vectors to be handled as complex numbers. Furthermore, real numbers are handled as subsets of complex numbers (vectors).



- Basic Outline

Since complex numbers are simply expansions of vectors in GRAPES, vector calculations may be used unchanged (except for dot products).

The x component of a vector corresponds to the real number part of the complex number, and the y component corresponds to the imaginary part.

In addition to corresponding to the four arithmetic operations for complex numbers, and integer multiples, exponential functions trigonometric functions, and hyperbolic functions are expanded to have complex number arguments.

- Complex Number Planes

Since 'points' on basic graphic elements and curves are vectors, they may be handled unchanged as 'Points on Complex Number Planes' in the Complex Number Mode.

An error is normally generated when real number values are passed to vector transformations of, for example, point P, however such points have meaning as points on a complex number plane in the Complex Number Mode.

Example: $P = 2$ generates an error in the Normal Mode.

$P = 2$ is equivalent to $P = (2, 0)$ in the Complex Number Mode.

9-8 Complex Number Calculations and Functions

- Arithmetic Operations and Exponentiation

Addition, subtraction, multiplication, division, and exponentiation are possible between complex numbers, and between complex numbers and real numbers.

☆ Multiplying two vectors (e.g., $P * Q$) returns a dot product, while in the Complex Number Mode it returns the product of complex numbers.

☆ Exponentiation permits integer multiplication of complex numbers and complex number multiplication of positive numbers. Squaring of positive complex numbers is based on Euler's formula $e^{i\theta} = \cos \theta + i \sin \theta$. Angles are limited to the circular method.

- Imaginary Number Units

The imaginary number i may be used. $i = (0, 1)$.

Example: $a + b i = a + b (0, 1) = (a, b)$

- Handling of $\sqrt{-1}$

Place a check in the [Handle $\sqrt{-1}$ as i] checkbox under the [Function] tab in the Options Window.

Sqrt (negative number) will return an error unless this is checked.

- Comparative Operators (equality, inequality)

Comparative operators may be used only with real numbers. Comparative operators cannot be used to compare complex numbers, or complex numbers and real numbers.

- Functions Expanded in the Complex Number Mode

exp, sin, cos, tan, sinh, cosh, tanh, sol

☆ Multi-value functions are not handled.

☆ Functions for vectors may be used unchanged.

☆ The sol function returning the solution to an equation is also able to handle imaginary numbers.

Example: $\text{sol}(x^4 = 16, 1) = -2$, $\text{sol}(x^4 = 16, 2) = 2$

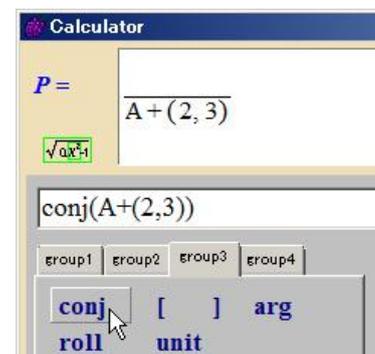
$\text{sol}(x^4 = 16, 3) = (0, -2)$, $\text{sol}(x^4 = 16, 4) = (0, 2)$

- Functions Returning Complex Conjugate Numbers

conj

Example: If $P = (1, 2)$, $\text{conj}(P) = (1, -2)$.

☆ The conj function is not normally provided in the scientific calculator, however in the Complex Number Mode the [conj] button is displayed under the [group 3] tab.



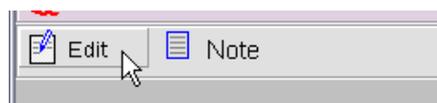
Chapter 10 Stickers and Labels

10-1 Editing Stickers

Stickers are used to add a description to a project. Stickers may display text as well as equations and equation values.

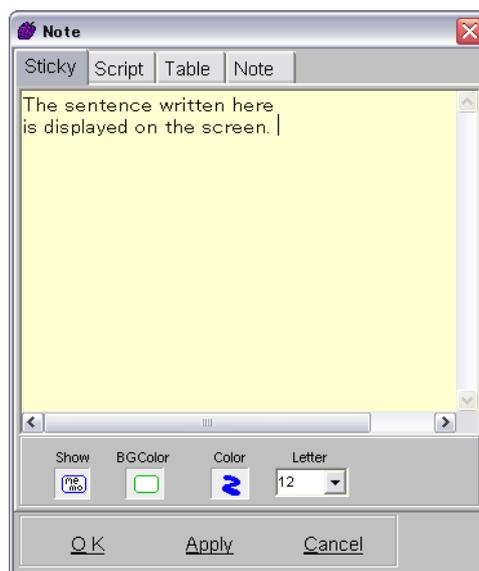
- Editing Stickers

1. Click on [Edit] in the Notes Area.



2. Enter the sticker in the Notes Window.

The typed text is displayed.



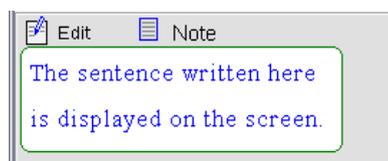
3. Select Text Color, Background Color, and Text Size.

4. Select the display method.

Select with/without border, with/without (transparent) background, or non-display.

5. Click on [OK].

Click on [Apply] to check entry.



- Sticker Characteristics

- a. Displayed details

Text, equations, equation values

- b. Text options

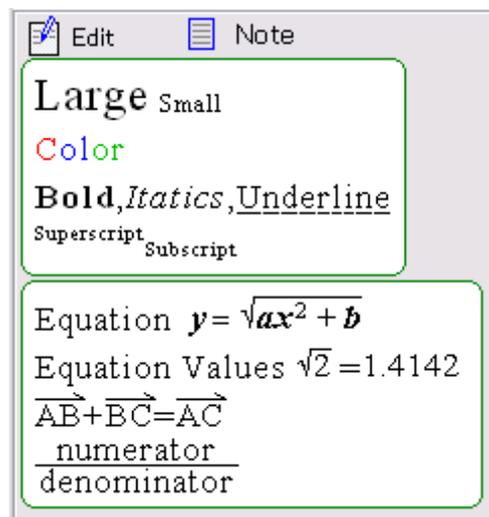
Text color, bold, italics, underline, text size, symbol font, superscript, subscript, vector symbols, fractions

- c. Text locations

Anywhere in the Graph Display Area, or in the Data Panel.

- d. Miscellaneous

Text may be displayed over a maximum of ten positions.



10-2 Moving Stickers

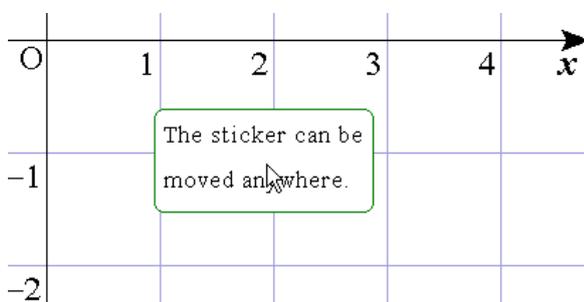
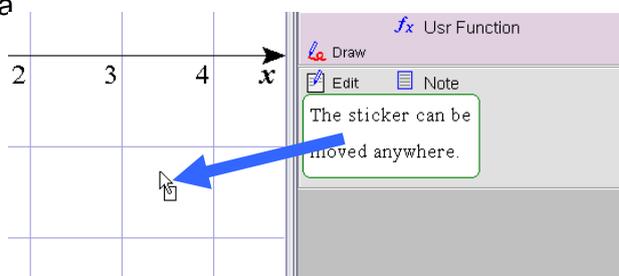
Stickers are created on the Data Panel, and may be moved to the Graph Display Area by dragging with the mouse.

- **Moving Stickers to the Graph Display Area**

1. Left-click and hold on the sticker.
2. Drag the sticker to move it.

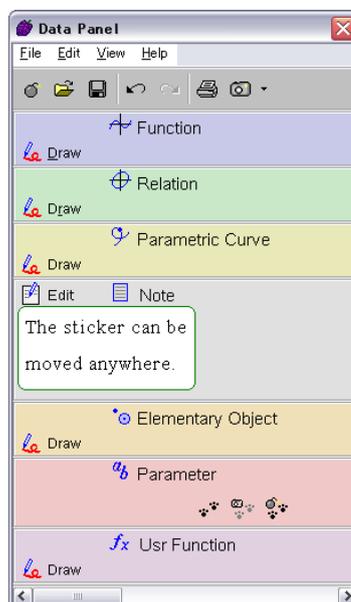
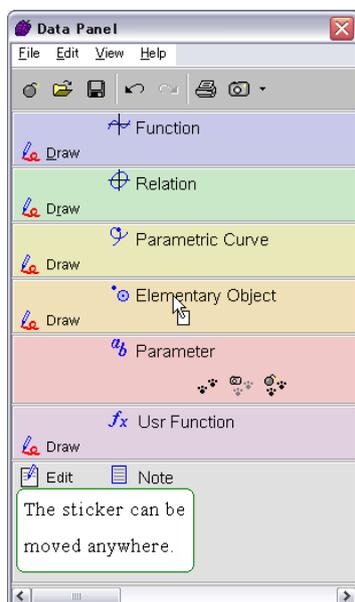
Release the left button.

The same procedure is used to move a sticker from the Graph Display Area to the Data Panel.



- **Moving a Sticker Inside the Data Panel**

A sticker or a title in the Notes Area may be dragged to any position within the Data Panel.



In addition to the Notes Area, a title may be moved up and down in any area in the Data Panel by dragging with the mouse.

10-3 Displaying Equations and Equation Values

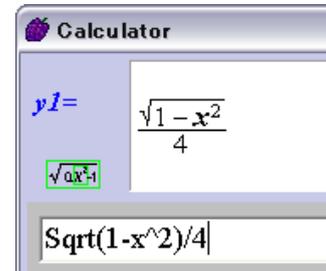
- Equation Text and Equation Images

An equation expressed as text is referred to as ‘equation text’, and its actual representation is referred to as the ‘equation image’.

In the scientific calculator example at right, ‘Sqrt(1-x^2)/4’ is the equation text, and

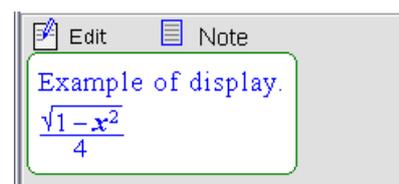
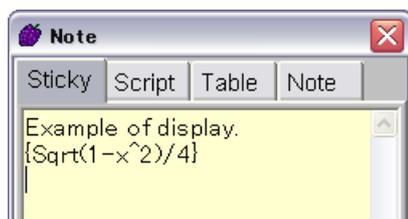
$$\frac{\sqrt{1-x^2}}{4}$$

is the equation image.



- Displaying the Equation Image

Enclose the equation text in half-width curly brackets ({}) to display the equation image in stickers and labels.



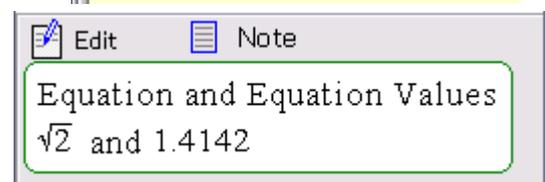
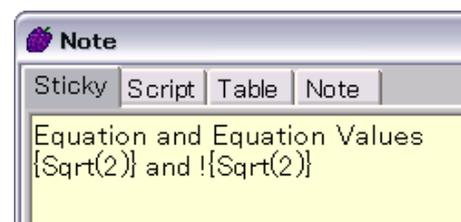
☆ To create equation text, simply create the equation in the scientific calculator, and copy the text.

- Displaying the Equation Values

The equation values are displayed when the equation text is enclosed in ‘!{}’.

Example: {Sqrt(2)} represents $\sqrt{2}$, and !{Sqrt(2)} represents the value of $\sqrt{2}$ (i.e. 1.4142).

In addition to constants, equation values including parameters and point coordinates may be displayed.



- Number of Decimal Places Displayed

Equation values are displayed in floating point values of up to five significant digits. The number of significant digits is changed by adding ‘| number of significant digits’ to the equation.

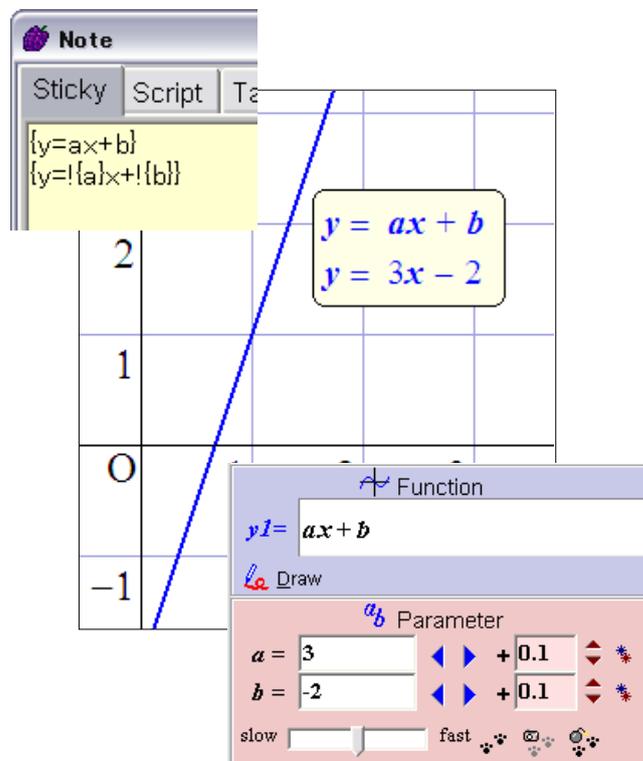
Example: !{Sqrt(a)|8} displays the value of \sqrt{a} as an eight-digit floating point value.

10-4 Displaying Changing Equations

- Displaying Equations With Changing Coefficients

When the values of parameters a and b are provided, the function equation including these parameters may be displayed using stickers. For example, $y = 3x - 2$ is displayed if $a = 3$ and $b = -2$ in the equation $y = ax + b$.

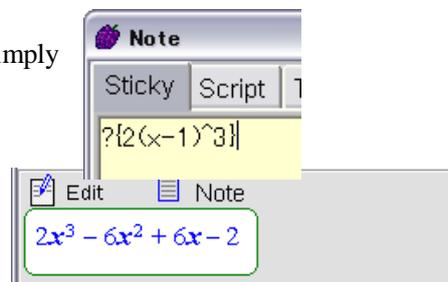
If the equation $\{y=ax+b\}$ is entered in a sticker, the equation $y = ax + b$ is displayed. $\{y=!{a}x+!{b}\}$ is used to display the values a and b . The '+' before $!{b}$ is automatically converted to '-' in accordance with the value of b .



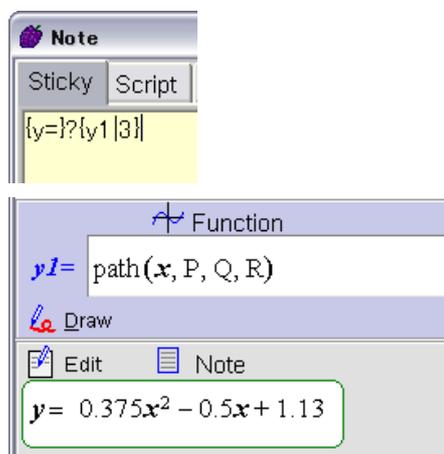
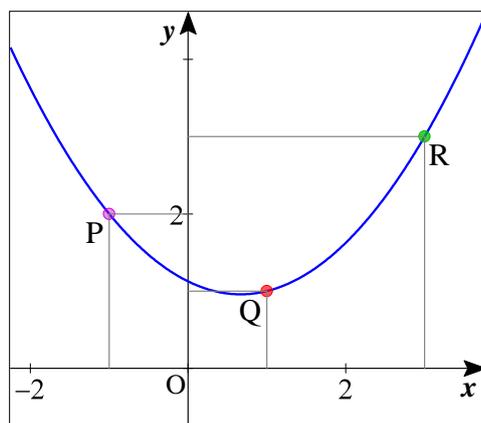
- Displaying Polynomial Expressions

Polynomial expressions of six dimensions or less may be simply displayed.

For example, $\{?\}$ is used to display the expansion of the function $2(x-1)^3$.



The diagram below shows the graph of the quadratic function $y1 = \text{path}(x, P, Q, R)$ passing through the three points P, Q, and R, and the equation.



10-5 Text Options

- Bold, Italics, Underline

- ◆ Bold

Text enclosed within `` and `` (or `` and ``) is displayed in bold.

- ◆ Italics

`<I>` and `</I>`

- ◆ Underline

`<U>` and `</U>`

- Text Size

- ◆ Large text

`<L>` and `</L>`

- ◆ Small text

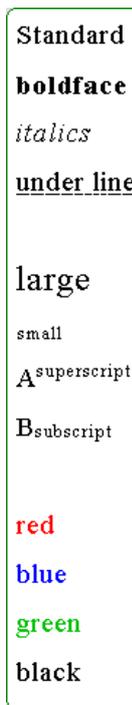
`<S>` and `</S>`

- ◆ Superscript text

`^{` and `}`

- ◆ Subscript text

`_{` and `}`



- Colored Text

Six colors (black, blue, green, red, purple, grey) are available for colored text.

Text enclosed within `<text color>` and `</text color>` is displayed in the specified color.

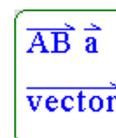
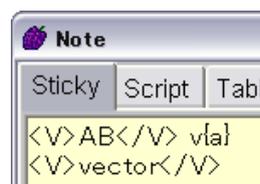
☆ Any color may be specified using `<color = #rrggbb>` and `</color>`. rrggbb are the hexadecimal values for the three primary colors. For example, red is specified with FF0000, and yellow with FFFF00.

- Returning to the Original Format

The original format is restored with `<Normal>`.

- Vector Symbols

`<V>` and `</V>` Same as `v{ text string }`.



- Symbol Font Format

`<Symbol>` and `</Symbol>`

Used for Greek lettering etc.

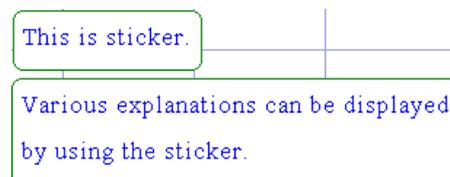
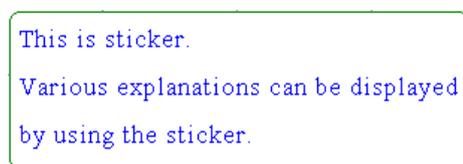
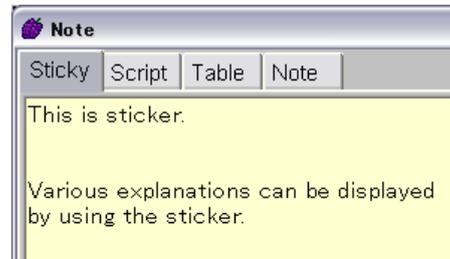
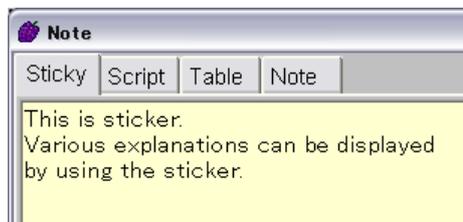
- Fractions

`<Frac>` - / - `</Frac>`

10-6 Displaying Multiple Stickers

- Displaying Multiple Stickers

Insert two or more lines of consecutive spaces at the point at which the text in the sticker is to be split.



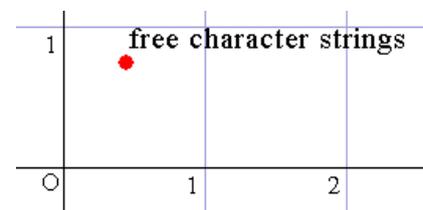
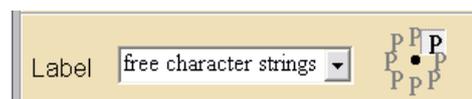
- ☆ Sticker background color and display method cannot be selected for individual stickers.
- ☆ Overall text size and text color cannot be selected for individual stickers. Use the Text Options above to select text color for each sticker.
- ☆ Up to ten stickers may be displayed.

10-7 Labels

As with stickers, text strings and equations may be displayed if the labels for basic graphic elements, curves, and linked graphic elements are used.

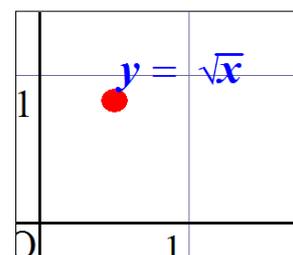
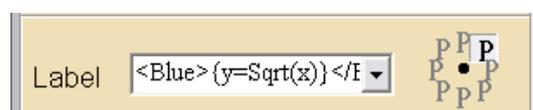
- Changing Label Text

Overwrite the label text.



- Text Options and Equations

Text options and equations may be entered in the same format used with stickers.



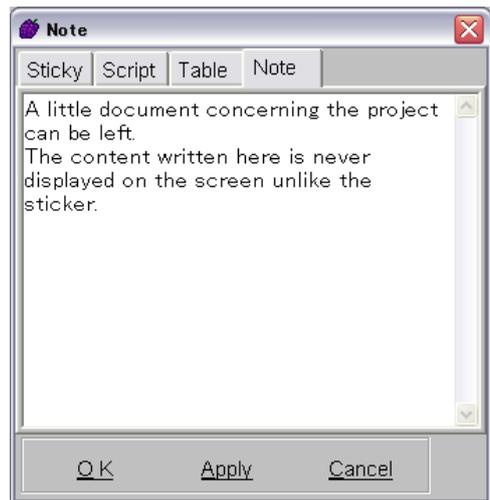
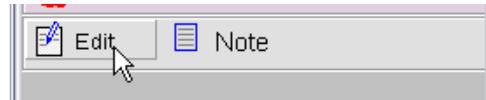
10-8 Notes

When you want to create a short document while creating a GRAPES project, select [Notes].

- Editing Notes

1. Click on [Edit] in the Notes Area.
2. Click on [Note] in the Notes Window.
3. Enter the required text.
4. Click on [OK].

The entered text is saved to a file. However it is not displayed on the screen as with stickers.



10-9 Font Configuration

After GRAPES 6.70, Sticky and Script can be dealt with without depending on kinds of languages. However, most fonts correspond to only limited languages except for a few kinds of Unicode font. For this reason, here users can select the font.

To Show Font Setting Window

Click on the option button of area palette to show the option window, and then click on “Font” tag.

- ◆ To Configure Fonts for Editing

Select a font for editing Sticky, Note and Script

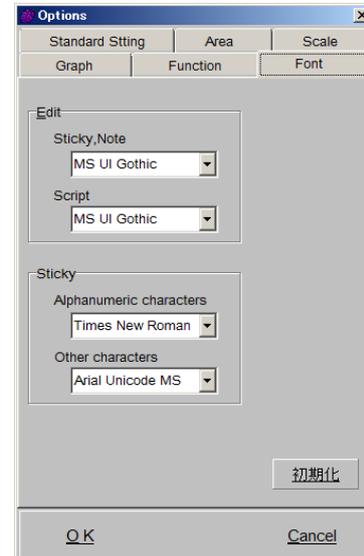
To Configure a Font for Sticky

You can select different fonts for Alphanumeric characters and for Other characters

- ◆ Return to Default of Fonts

If you click on “Default”, the regular font of the system for editing, Times New Roman for Alphanumeric characters, MSP Gothic for Other characters of Sticky, will be set respectively.

Since the fonts configured here are recorded in each user’s registry when finishing GRAPES, you can use it with same configuration next time. However, the change of configuration is not recorded in respective project file.



Chapter 11 Tables

11-1 Data Input

GRAPES provides a 200 row x 10 column table. Values in cells in the table may be referenced from functions outside the table.

- Data Input

1. Click on [Edit] in the Notes Area.
2. Click on [Table] in the Notes Window.
3. Enter data in a cell.

- Only numerical values may be entered.
- The results of the calculation become the cell value when a function equation is entered. Only function equations recognized by GRAPES may be entered. See 'Equation Text and Equation Image' for details.

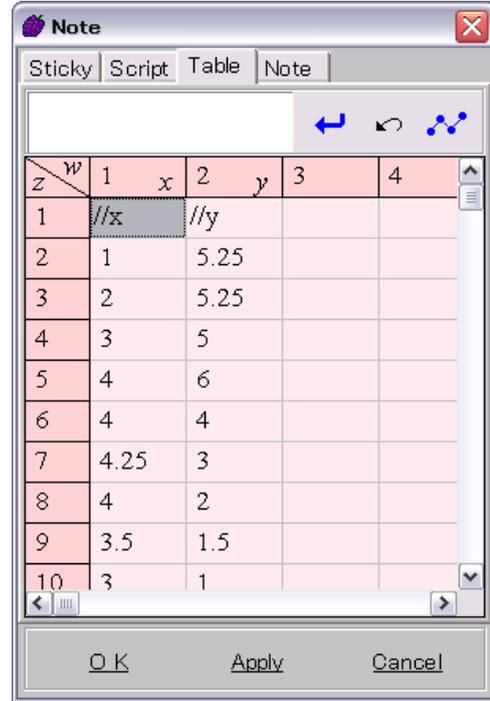
The y coordinate is entered in the adjacent cell to the right when the calculation result is two-dimensional data.

- Text strings beginning with '//' or '' are displayed as comment text.
- Data is deleted with the [Del] key.
- Double-clicking in the Edit Mode allows the use of the scientific calculator to enter values in cells.
- When using functions and the scientific calculator, 'z' represents the row number, 'w' the column number, 'x' the value of the cell in the first column of the same row as the edited cell, and 'y' the value of the cell in the second column of the same row as the edited cell.

Example: When the edited cell is the third column of the second row as in the diagram at right;

$$z = 2, w = 3, x = 2.5, y = 6$$

4. Click on [OK].



z \ w	1	x	2	y	3	4
1	1.5		5			
2	2.5		6	=x+y		
3	3.5		7			
4	1.5		8			
5						

Click on [Apply] to check entry.

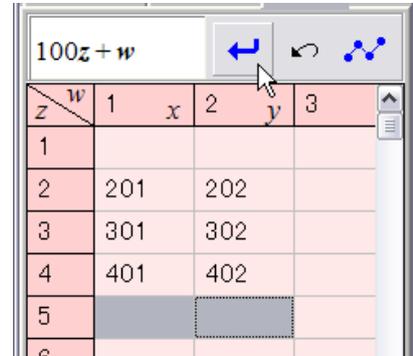
☆ Table data is not entered until [Apply] or [OK] are clicked. When an existing graph references table data, the graph is not updated until these buttons are clicked to change the data.

- Batch Calculations 

Batch calculations are executed for the selected area.

1. Click on the equation entry window at the top of the table to enter a function equation.

- In the equation, 'z' represents the row number for each cell in the block, and 'w' represents the column number. Furthermore, 'x' represents the value of the cell in the first column of the same row as each cell, and 'y' represents the value of the cell in the second column of the same row.
- Cells(row,column) is used to reference other cells.

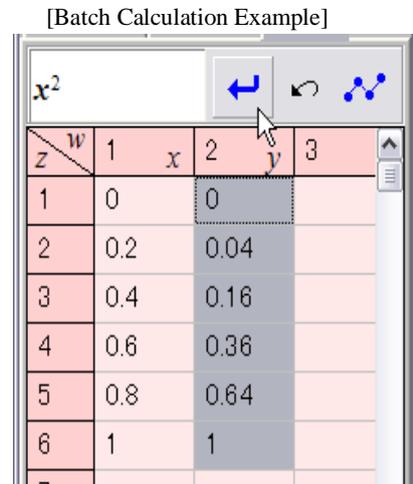


100z + w					
z \ w	1	x	2	y	3
1	1	x	2	y	3
2	1	201	202		
3	3	301	302		
4	4	401	402		
5					

2. Click on the  button (Calculate).

- Calculation progresses in the row direction from top-left to bottom-right of the selected area.
- The y coordinate is entered in the adjacent cell to the right when the calculation result is two-dimensional data.
- When calculation is complete, the edited cell (selected area) is immediately below the selected area.

[Batch Calculation Example]



x ²					
z \ w	1	x	2	y	3
1	0	x	0		
2	0.2		0.04		
3	0.4		0.16		
4	0.6		0.36		
5	0.8		0.64		
6	1		1		

- Moving the Edited Cell

- Move with the cursor.
- Move to the right cell with the [Tab] key, and to the left cell with the [Shift]+[Tab] keys.
- Move to the cell below with the [Enter] key.
- Move to the desired cell by clicking with the mouse.

11-2 Editing Tables

- Selecting Rows and Columns

Click on the row and column numbers.

Click on the cell at top-left to select the entire table.

- Selecting Blocks

Drag with the mouse, or move with the [Shift]+cursor keys.

☆The mouse cannot be used for block selection when the cell is in the Edit Mode. In this case, press the [ESC] key to clear the Edit Mode.

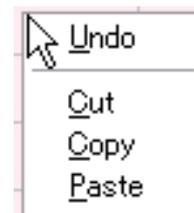
- Moving Row and Column Data

Drag the row and column numbers.

- Copy and Paste

Select from the right-click pop-up menu.

- Shortcut keys (copy with [CTRL]+[C], cut with [CTRL]+[X], paste with [CTRL]+[V]) may also be used.
- Copy and paste with Excel spreadsheets is also possible.



- Undo 

Click on the  button (Undo).

- Change Column Width

Drag the boundary of the top-most cell.

11-3 Using Table Data

- Reading Data

- ◆ Cells(row,column): Value of specified cell

Both constants and equations may be assigned to rows and columns.

An error value is returned if an invalid cell location, or a cell with no value, is specified.

Cell values are real numbers or error values.

- ◆ CellsP(row, column) : Vector consisting of values of an assigned cell and of the right-hand side cell.

- Creating Scatter Diagrams 

Draws a scatter diagram within x and y coordinates in two rows of table data.

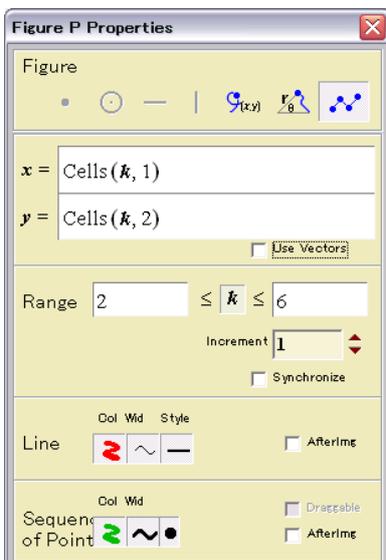
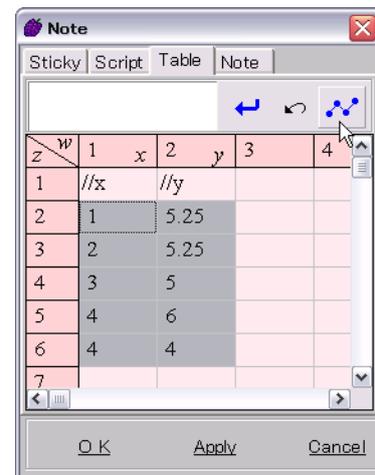
1. Select the range of cells to be graphed.
2. Click on the  button (Scatter chart).

The Create Scatter Diagram Window is displayed.

3. Remove the check for Point Sequence or Broken Line as necessary.

A graph is drawn only if at least one is selected.

The data range selected in 1. above may be changed here if necessary.



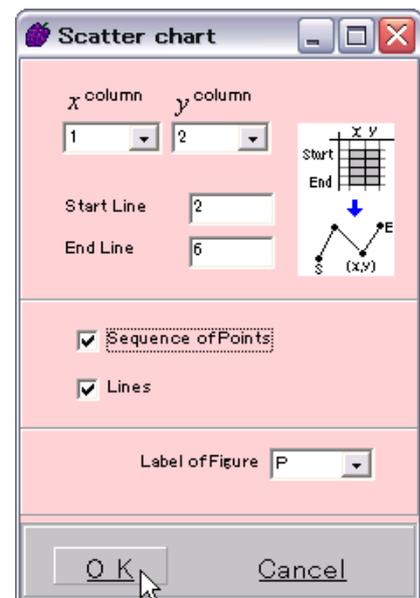
4. Specify the graphic element name.

The element is drawn using Point Sequence in Curve.

5. Click on [OK].

The Curve Property Screen is displayed.

6. Set point and line color and thickness, and click on [OK].



☆ A graph created in this manner references a table, however table

data currently being edited is not reflected in the graph. Click on [Apply] at the bottom of the Notes Window to ensure that data currently being edited is reflected in the graph.

11-4 Scripts and Table Data

The use of scripts allows substitution of series of calculation results (e.g. number sequences) in cells.

- Substitution in Cells

Cells(row,column) := equation (both constants and equations may be entered in rows and columns)

Example: Cells(1,2) := 4

★ When the value at right is two-dimensional data, the values are substituted in the specified cell and the adjacent cell to the right.

For example, Cells(1,2) := (4,5) produces the same results as Cells(1,2) := (4) and Cells(1,3) := (5).

In case the value at right is two-dimensional data, it is also possible to use CellsP(row, column)=equation.

- Initialize All Cells

ClrAllCells

Deletes values in all cells in the table.

- Samples

Substitute the 1st to the 20th items of a Fibonacci sequence in a cell.

Cells(1 , 1) := 1

Cells(2 , 1) := 1

For n := 3 to 20

Cells(n , 1) := Cells(n-1 , 1) + Cells(n-2 , 1)

Next

The column number may be omitted in Cells(row,column). In this case, the column number is assumed as 1.

Cells(row) = Cells(row, 1)

If this format is used, the script above is as follows.

Cells(1) := 1

Cells(2) := 1

For n := 3 to 20

Cells(n) := Cells(n-1) + Cells(n-2)

Next

See 'Chapter 14 Scripts' for further details.

Chapter 12 Mastering the Use of Functions

12-1 Detailed Settings for Graphs

- Displaying the Graph Setup Window

Click on the  button (Options) on the Area Pallet to display the Options Window, and click on the [Graph] tab.

- ◆ Display Linked Graphic Elements

If a check is placed in this checkbox, the linked graphic element is not displayed if one or points comprising the linked graphic element are non-display.

- ◆ Draw While Dragging

When dragging a point, the graph referencing that point is normally redrawn after dragging is complete, however if a check is placed in this checkbox, it is drawn while dragging. Drawing may be slow due to program limitations.

- ◆ Draw Relation Graphs While Calculating

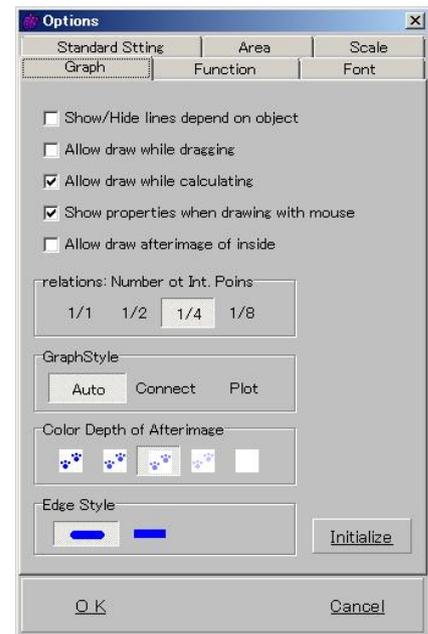
Relation graphs are drawn while calculating to save time. If this check is removed, the graph is drawn when calculating is complete, and flicker is therefore eliminated.

- ◆ Display Properties After Drawing With Mouse

The Properties Screen is opened by default when drawing by dragging points and line segments with the mouse. If this check is removed the screen remains closed.

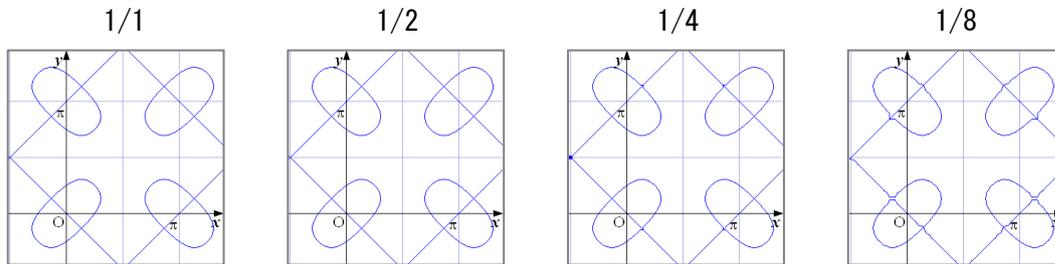
- ◆ Leave an Image of the Inside of the Graphic Element

Inside painting of graphic elements (e.g. circles, curves) is not left as an image by default. If a check is placed in this checkbox, inside paint is also left as an image.



- ◆ Calculated Density of Relations

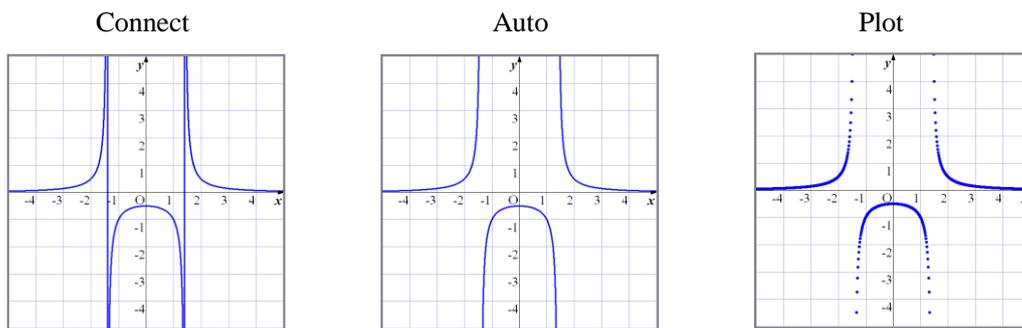
With most relations, the function values for all points in the graph display area are investigated to draw the graph. Some functions require considerable time for calculation, and the density of points in the calculation is therefore reduced. See '4-4 How to Draw Relations'.



▲ Graph of $\sin x + \sin y = \sin 3x + \sin 3y$

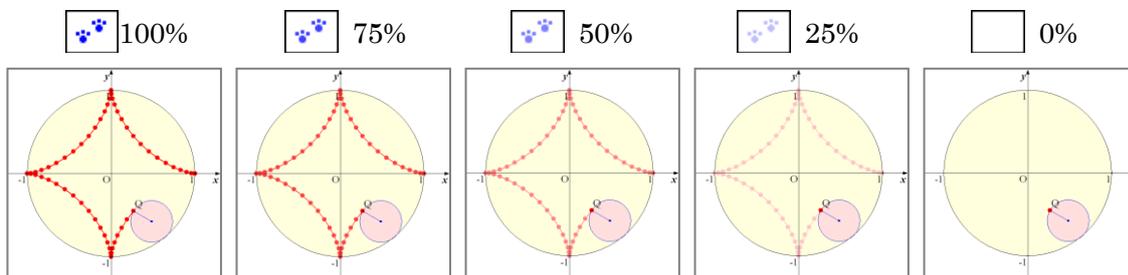
- ◆ Methods of Drawing Explicit Functions

Function graphs are drawn by calculating the value of y in relation to x . When drawing the graph, it is possible to select whether to draw a graph joining all these points, simply plot the points, or join the points only under a given set of conditions.



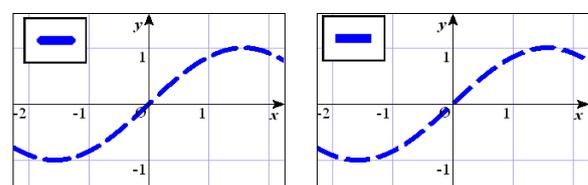
- ◆ Afterimage Density

By default, afterimages and locuses are displayed in a lighter shade of the graph color, however the shade may be changed.



- ◆ Line Edge Processing

The shape of the end of the lines may be specified when a graph is drawn with dotted or broken lines.



12-2 Detailed Settings for Functions

● Displaying the Function Setup Window

Click on the  button (Options) on the Area Pallet to display the Options Window, and click on the [Function] tab.

◆ Allow Negative Radius Vector

Normally the number of polar equation radius vectors and circle radii of 0 or greater, however allowing negative radius vectors and radii has many advantages.

Negative radius vectors and radii are allowed at initial setup, however this may be changed.

◆ Handle Vectors as Complex Numbers

When a check is placed in this checkbox, vectors are handled as points on complex number planes. See ‘9-7 Vectors and Complex Numbers’ for details.

◆ Θ in Defined Functions and Relations Handled as Angular Displacement

θ in relations and defined functions is expressed as angular displacement of the point (x,y) . Note that θ functions as a parameter in functions and basic graphic elements. Remove this check to handle θ as a parameter in both relations and defined functions.

◆ Handling of log

The user is able to select whether the log function is handled as a natural logarithm, or as a common logarithm. Set as a natural logarithm at initial setup. The log function is not available in the scientific calculator, however the natural logarithm function \ln is available.

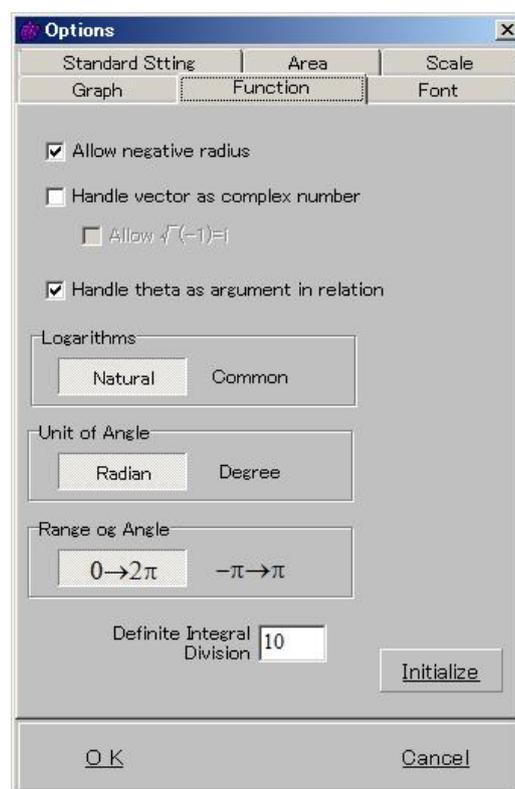
◆ Angular Units

The user is able to select whether angles are handled with the circular method or the frequency method. This selection affects trigonometric functions, inverse trigonometric functions, and the arg function. The circular method is selected at initial setup.

◆ Range of Angles

Returns the range of the angles obtained with inverse trigonometric functions and the arg function. Set to $0 \leq \theta < 2\pi$ at initial setup.

This setting also affects display of the linked graphic element ‘angle’. When the range of the angle is $-\pi \leq \theta < \pi$, ‘angle’ always displays the minor arc.



- Definite Integral Function \int (integration variables, top limit, bottom limit, equations) Partition Size

Definite integrals employ the 4th order approximation equation. This approximation equation does not result in logical errors in up to and including 9th order polynomial expressions, and its application to smaller individual sections of the integral partition allows for very high calculation accuracy for general functions.

12-3 GRAPES Functions

● Left Functions

Arguments are normally on the right-hand side of the function. The parentheses indicating the range of the arguments may be omitted with trigonometric functions and logarithmic functions. For example, $\sin(2x)$ may be written as $\sin 2x$. Other functions require that the range of the argument be expressed clearly in parentheses.

- ♦ $\sin x, \cos x, \tan x$: Trigonometric functions
- ♦ $A \sin x, A \cos x, A \tan x$: Inverse trigonometric functions

Angular units for trigonometric functions and inverse trigonometric functions are set to the circular method at initial setup. The circular method or the frequency method may be selected under the [Function] tab in the Options Window.

- ♦ $\exp(x)$: e^x
- ♦ $\log x$: Natural logarithm or common logarithm

Set to natural logarithm at initial setup. Change to the frequency method under the [Function] tab in the Options Window.

- ♦ $\log(a, x)$: Log x with base a .
- ♦ $\ln(x)$: Natural logarithm
- ♦ $\sinh x, \cosh x, \tanh x$: Hyperbolic functions
- ♦ $\sqrt{x}, \sqrt[3]{x}$: Square root, cube root, (entered from keyboard as 'sqrt' and 'cbrt').
- ♦ $\text{int}(x)$: Integer portion (equivalent to Gauss symbol)
- ♦ $\text{round}(x)$: Rounding
- ♦ $\text{frc}(x)$: Fraction $\text{frc}(x) = x - \text{int}(x)$
- ♦ $|x/$ or $\text{abs}(x)$: Absolute value ('| |' entered from keyboard as '[]')
- ♦ $\text{unit}(x)$: Sign
- ♦ $\text{rnd}(x)$: A random integer of 0 or greater and less than x .

Note that a random real number between 0 and 1 is returned when $x = 1$.

- ♦ $\Gamma(x), B(x, y)$: Gamma and Beta functions
- ♦ $f(x), g(x), h(x), f1(x) \sim f5(x)$: Defined functions
- ♦ $f'(x) \sim f5'(x)$: Derivatives of defined functions.
- ♦ $f''(x) \sim f5''(x)$: 2nd derivatives of defined functions.
- ♦ $F(x) \sim F5(x)$: Indefinite integrals of defined functions. $F(x) = \int_0^x f(t) dt$

Note that $f(x)$ is limited to up to and including 8th order polynomial expressions.

- ♦ $nCr(x, y)$: Binomial coefficient

- $\text{gcd}(x, y)$, $\text{lcm}(x, y)$: Maximum and minimum common multiples.
- $f(x, y)$, $f(x, y, z)$, $f(x, y, z, w)$: Defined functions (similar to $g, h, f1 \sim f5$)
- $\text{Max}(x_1, \dots, x_n)$, $\text{Min}(x_1, \dots, x_n)$: Maximum and minimum values.

Up to 20 arguments may be used, separated by commas.

- $\text{Cells}(m, n)$: Read table data. The value in the cell at row m and column n .

An error value is returned if an invalid cell location, or a cell with non-numerical data, is specified.

The value of the first column is returned if the column number is omitted. $\text{Cells}(m) = \text{Cells}(m, 1)$

- $\text{rgb}(r, g, b)$: Returns a 24-bit integer color value.

r , g , and b are real numbers between 0 and 1 representing the strength of red, green, and blue.

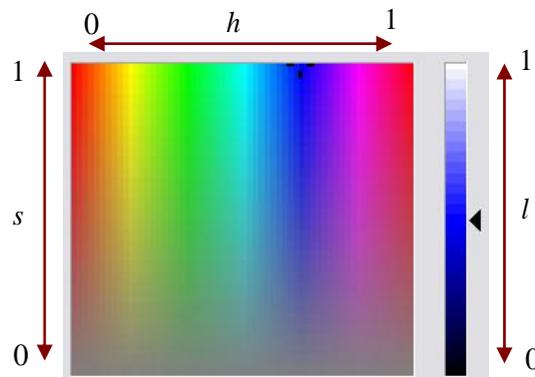
Example: $\text{rgb}(1,0,0)$ represents red, $\text{rgb}(1,1,0)$ represents yellow, and $\text{rgb}(0.5,0.5,0.5)$ represents gray.

- $\text{hsl}(h, s, l)$: Returns a 24-bit integer color value.

h , s , and l are real numbers between 0 and 1 representing hue, chroma, and brightness.

The function is periodic for h . $\text{hsl}(h+1, s, l) = \text{hsl}(h, s, l)$

Example: $\text{hsl}(0,1,0.5)$ represents red, $\text{hsl}(0,1,0.8)$ represents light red, and $\text{hsl}(0,0,0.5)$ represents gray.



- $\text{area}(x)$: Display area

Top-left of the display area is ($\text{area}(1)$, $\text{area}(3)$), bottom-right is ($\text{area}(2)$, $\text{area}(4)$).

- $\text{Pressed}(\text{virtual key code})$: Checks whether or not the specified key has been pressed.

True (1) if the key returned with the virtual key code has been pressed, and false (error value) if not.

The virtual key code is an integer defined in the Window system.

12-4 Special Functions

- Right Functions

Functions in which the function symbol is placed after the argument.

- ◆ $x!$: Factorial
- ◆ x° : Converts frequency to radians ('°' entered from keyboard as 'degree').
- ◆ $.x, .y$: x and y components of a vector.
- ◆ $.r$: Circle (basic graphic element) radius

- Functions Without Arguments

- ◆ $y1 \sim y20$: Function values (may only be referenced from within explicit and relations)
- ◆ P, Q etc.: Points
- ◆ O: Origin
- ◆ X: Variable sets (x, y)
- ◆ r, θ : $r = \text{len}(x, y)$, $\theta = \text{arg}(x, y)$ (usable only within relations and defined functions)

- Functions and Functions and Parameter Notation as Arguments

- ◆ $\text{sol}(f(x) = g(x), n)$: Within the solutions to a 4th order or lower equation $f(x) = g(x)$, returns the n th from left of the real number parts in order of smallest to largest ($n = 1, 2, 3, 4$).
 $n = 1$ when n is omitted.

- ◆ $\text{cof}(n, f(x))$: Returns the n th coefficient when $f(x)$ is a 6th order or lower polynomial.

Example: $\text{cof}(2, (x-2)^3) = -6$

- ◆ $\Sigma(\text{parameter}, \text{initial value}, \text{final value}, \text{equation})$: Sum of sequence of numbers ('Σ' entered from keyboard as 'sum').

Example: If $\sum_{k=1}^5 k^2$, $\Sigma(k, 1, 5, k^2)$.

An equation including the variable x in the initial or final value cannot be substituted.

- ◆ $\Pi(\text{parameter}, \text{initial value}, \text{final value}, \text{equation})$: Product of sequence of numbers ('Π' entered from keyboard as 'product').

An equation including the variable x in the initial or final value cannot be substituted.

- ◆ \int (integration variable, bottom limit, top limit, equation): Definite integral ('∫' entered from keyboard as 'igr').

Example: If $\int_0^\pi \sin 2x \, dx$, $\int(x, 0, \pi, \sin 2x)$.

Parameters and x are used as the integration variable.

An equation including the variable x in the upper or lower limit cannot be substituted.

- ♦ Step(parameter): Range of increase/decrease for parameter.

12-5 Vector Functions

- Real Number Value Functions

- ◆ $\text{path}(x, P_1, \dots, P_n)$: An $(n-1)^{\text{th}}$ function passing through the provided n number of points.
- ◆ $|P|$ or $\text{len}(P)$: Vector size ('|'| entered from keyboard as '[']').
- ◆ $\text{len}(P_1, P_2)$: Distance between two points.
- ◆ $\text{arg}(P)$: Angular displacement (the range of the angle is set in [Function Setup] ($0 \leq \text{arg} < 2\pi, -\pi \leq \text{arg} < \pi$)).
- ◆ $\text{arg}(P_1, P_2)$: Angle $\angle P_1OP_2$
 $\text{arg}(P_1, P_2, P_3)$: Angle $\angle P_1P_2P_3$ forming three points.
- ◆ $\det(a, b, c, d)$: Determinant value $ad - bc$
 $\det(P, Q)$: Determinant value $P.x \cdot Q.y - P.y \cdot Q.x$
- ◆ $\text{perp}(P, A, B)$: Length of perpendicular line drawn from point P to straight line AB (length of perpendicular line entered from keyboard as 'perp')
- ◆ $\text{Crad}(A, B, C)$: Radius of the circumcircle of the triangle ABC ('circumcircle radius' entered from keyboard as 'crad').
- ◆ $\text{Irad}(A, B, C)$: Radius of the incircle of the triangle ABC ('incircle radius' entered from keyboard as 'irad').

- Vector Value Functions

- ◆ $\text{roll}(t)$: Returns points $(\cos t, \sin t)$ on the unit circle.
- ◆ $P(n)$: Coordinates of the nth point (points P, Q, ..., V, A, ..., N are numbered 1, 2, 3, ..., 21).

This function is used only in stickers and labels.

- ◆ $\text{unit}(P)$: Unit vector in the \overrightarrow{OP} direction.
- ◆ $\text{gcentr}(A, B, C)$: Center of gravity of the triangle ABC ('center of gravity' entered from keyboard as 'gcentr').
- ◆ $\text{ccentr}(A, B, C)$: Circumcenter of the triangle ABC ('circumcenter' entered from keyboard as 'ccentr').
- ◆ $\text{hcentr}(A, B, C)$: Orthocenter of the triangle ABC ('orthocenter' entered from keyboard as 'hcentr').
- ◆ $\text{icentr}(A, B, C)$: Incenter of the triangle ABC ('incenter' entered from keyboard as 'icentr').
- ◆ $\text{ecentr}(A, B, C)$: Excentre of triangle ABC ('Excentre entered from keyboard as 'ecentr')

Other excentres are $\text{ecentr}(B, C, A)$ and $\text{ecentr}(C, A, B)$

- ♦ $\text{intr}(A, B, C, D)$: Intersection of the two straight lines AB and CD ('intersection' entered from keyboard as 'intr').
- ♦ $\text{intr}(A, B, C, r)$: Intersection of the straight line AB and the radius r from the center C.
The other intersection is $\text{intr}(B, A, C, r)$.
- ♦ $\text{intr}(A, a, B, b)$: Intersection of the radius a from the center A and the radius b from the center B.
The other intersection is $\text{intr}(B, b, A, a)$.
- ♦ $\text{perp}(P, A, B)$: Base of the perpendicular line from the point P to the straight line AB ('base of the perpendicular line' entered from keyboard as 'perp').
- ♦ $\text{mid}(P, Q, m, n)$: The point dividing the line segment AB into $m : n$.
- ♦ $\text{rot}(P)$: Rotate point P around the origin by angle 90° ('rot' entered from keyboard as 'rot').
- ♦ $\text{rot}(P, t)$: Rotate point P around the origin by angle t .
- ♦ $\text{rot}(P, C, t)$: Rotate point P around point C by angle t .
- ♦ $\text{polygon}(t, P_1, P_2, \dots, P_n)$: A point on an edge of a polygon of n angles. One revolution of the polydon with $0 \leq t < n$ ($n \leq 20$).
- ♦ $\text{proj}(x, y, z, s, t, d)$: Returns the coordinates of a point in space (x, y, z) when projected onto a flat plane.
 s , t , and d are the plane angle, the solid angle, and the distance from the origin to the viewing point.
Parallel projection if $d = 0$ or d is omitted.
- ♦ See '9-6 Defined Functions and Vectors' for the relationship between defined functions and vectors.

12-6 Operators and Variables

● Operators

- ◆ $+$, $-$, \times , \div : Four arithmetical operators
- ◆ \wedge : Square of a real number

$$x \wedge y = x^y$$

If $x < 0$, a value results only if y is an integer.

Interpreted as the square of the function when placed immediately after a trigonometric function (e.g., $\sin^n x = (\sin x)^n$).

- ◆ $x \bmod y$: Integer remainder
- ◆ $>$, \geq , $<$, \leq , $=$, \neq Comparison operators

These operators return 1 if true, and an error value if false.

The graph is not drawn if the function value is an error value.

For example, for $y = x(x \geq 0)$, the graph is drawn only within the range $x \geq 0$.

- ◆ AND, OR, NOT: Logical operators

These operators return 1 if true, and an error value if false.

● Vector Operators

- ◆ $+$, $-$: Vector addition and subtraction
- ◆ \cdot : Vector dot product
- ◆ \times : Real number \times vector (x may be omitted)
- ◆ \div : Vector / real numbers

● Parameters

- ◆ $a, b, c, d, k, m, n, p, q, s, t, u, v, \theta$

θ is handled as a function expressing angular displacement with relations and defined functions.

● Variables

- ◆ x, y, z, w : z and w are used only with defined functions and scripts.
- ◆ θ : Angular displacement for polar equations (used only with relations and defined functions).
- ◆ X : Represents the point (x, y) .
- ◆ Y : Represents the point (z, w) (used only with defined functions and scripts).

● Constants

- ◆ e : Natural log base
- ◆ π : Circle ratio

12-7 Evaluation Priority Sequence

- Priority Sequence
 - ◆ The part enclosed in parentheses is evaluated first.
 - ◆ Functions and operators are evaluated in the following priority sequence.
 1. Right functions
 2. Exponent
 3. Multiplication with the multiplication symbol omitted
 4. Left Functions
 5. Multiplication and division
 6. Addition and subtraction
 7. Comparison operators, logical operators (AND has priority over OR)
 - ◆ Evaluated from left of the equation in all other cases.

- Functions in Which Arguments are Not Enclosed in Parenthesis

Arguments for the following functions must be enclosed in parentheses.

$$\sin, \cos, \tan, \operatorname{Asin}, \operatorname{Acos}, \operatorname{Atan}, \log, \sqrt{\quad}, \sqrt[3]{\quad}$$

The range of the arguments for these functions is determined as follows.

- ◆ Until an arithmetic operator, logical operator, comparison operator, space, or left function is encountered.

Example: $\sin a \cos x^2 + 1 = (\sin a)(\cos x^2) + 1$

$$\sin a(x+1)^2 + 1 = \sin(a(x+1)^2) + 1$$

$$\sin a(x+1)^2 + 1 = (\sin a)(x+1)^2 + 1$$

- ◆ All up to an exponent symbol placed immediately after a trigonometric function is handled as a single function.
- ◆ Right functions and exponent arguments are limited to immediately previous notation, values, and parentheses.

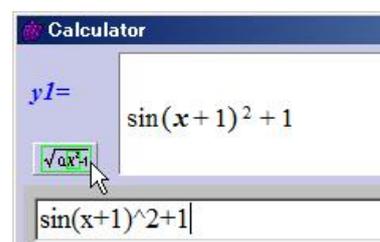
Example: $2n! = 2(n!), 2ax^2 = 2a(x^2)$

☆ Add parentheses, or delineate with spaces, if priority relationships are unclear.

- Displaying the Range of Functions Arguments

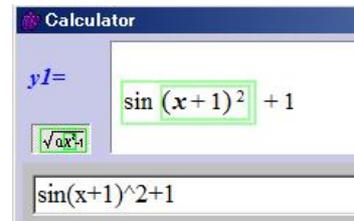
The above rules are in daily use, however they tend to be vague.

The manner in which GRAPES makes evaluations may be verified with the scientific calculator.



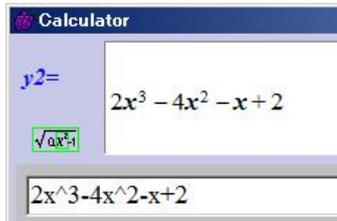
Click on the  button (Scientific Calculator Argument Display Range).

The range of the argument is displayed with a green boundary.

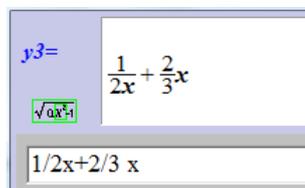


12-8 Function Input Examples

(1) $y = 2x^3 - 4x^2 - x + 2$

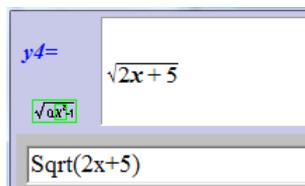


(2) $y = \frac{1}{2x} + \frac{2}{3}x$

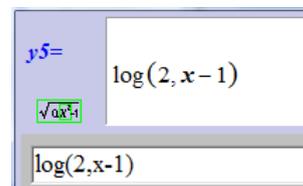


← Insert a space between 2 / 3 and x.

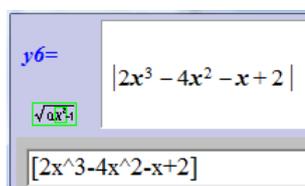
(3) $y = \sqrt{2x+5}$



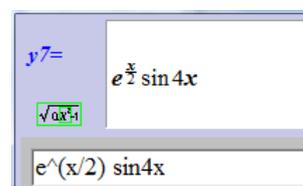
(4) $y = \log_2(x-1)$



(5) $y = |2x^3 - 4x^2 - x + 2|$



(6) $y = e^{\frac{x}{2}} \sin 4x$

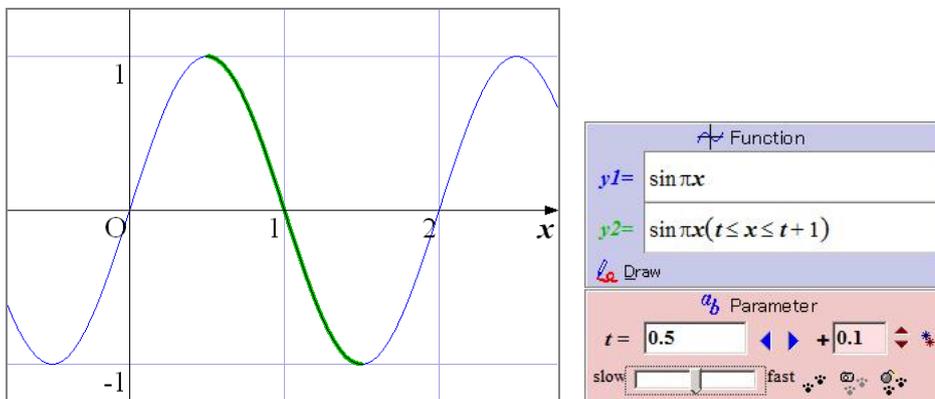


12-9 Limits to Domains

- Limits to Domains

The domain is placed after the function equation to limit the domain of the function.

Example: $y = x^2 (x \geq 0)$, $y = \sin \pi x (t \leq x \leq t+1)$

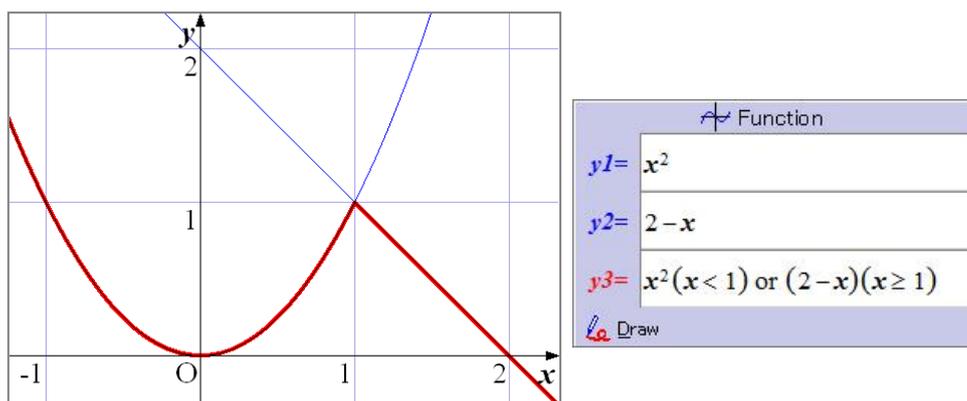


$t \leq x \leq t+1$ may be written as $(t \leq x)$ and $(x \leq t+1)$.

- Synthesis of Functions with Limited Domains

When two or more functions with limited domains are combined as in $y = \begin{cases} x^2 & (x < 1) \\ 2-x & (x \geq 1) \end{cases}$, they are

joined with the logical OR operator as in $y = x^2 (x < 1)$ or $(2-x)(x \geq 1)$.



The example above may be written as $y3 = y1(x < 1)$ or $y2(x \geq 1)$.

12-10 Derived Functions and Indefinite Integrals

To use differentiation and integration in GRAPES, first register the target function as a defined function, and use the derived function and indefinite integral.

- Using Derived Functions

1. Register the function to be differentiated as a defined function.

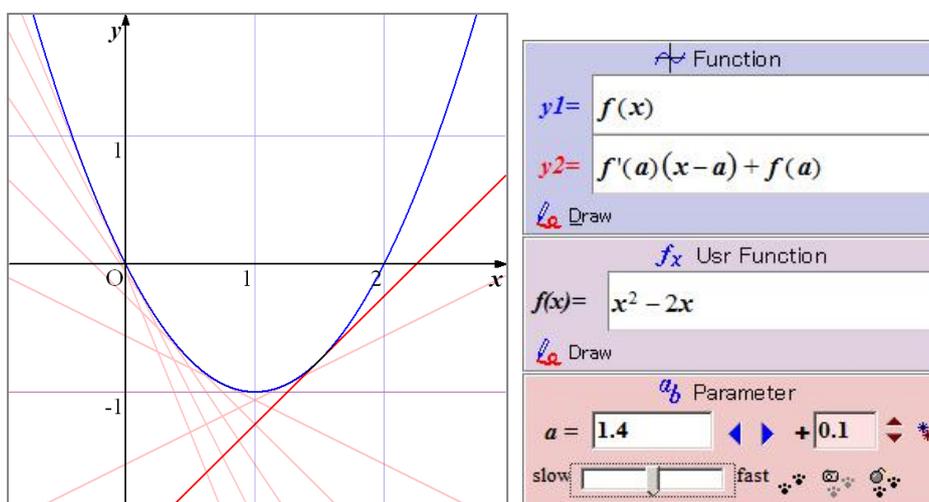
Register the function for differentiation as a defined function.

For example, as $f(x) = x^2 - 2x$.

2. Use the derivative of the defined function to write the graph equation.

Use the derived function (e.g. f' , g' , h') of the defined function to write the equation.

For example, $y2 = f'(a)(x - a) + f(a)$.



☆ First and second derivatives may be used.

- Using Indefinite Integrals

Eighth and lower order integer functions may be integrated. Use the Definite Integral Window and definite integral functions for integration of other functions.

1. Register the function to be integrated as a defined function.

For example, $f(x) = x^2 - 2x$.

2. Use the integral of the defined function to write the graph equation.

Use the integral function (e.g. F, G, H) of the defined function to write the equation.

$F(x)$ is expressed as $\int_0^x f(t) dt$.

Chapter 13 Looking at Functions

13-1 Displaying Coordinates and Functions

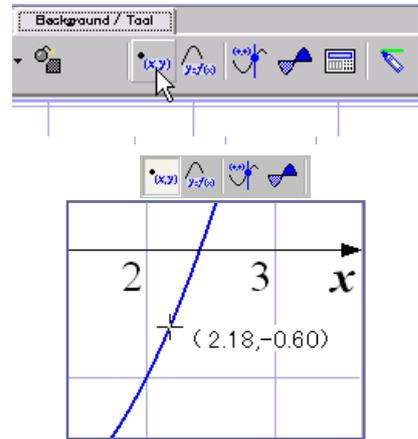
- Displaying Graph and Locus Coordinates

1. Click on the  (Show Coordinates) button on the Tools Pallet.

Click on the Display Coordinates button.

2. Move the cursor over the graph.

The coordinates of the point are displayed when the cursor is moved over the graph.



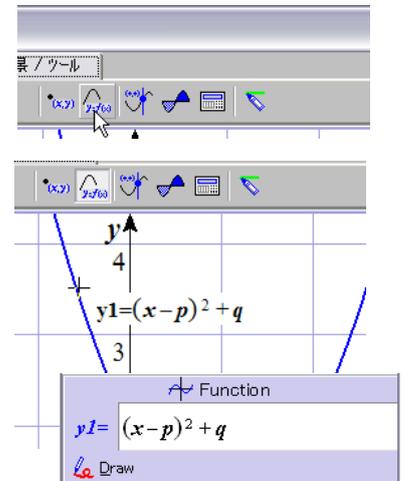
- Displaying Function Equations

1. Click on the  (Show Expression) button on the Tools Pallet.

Click on the Function Equation button.

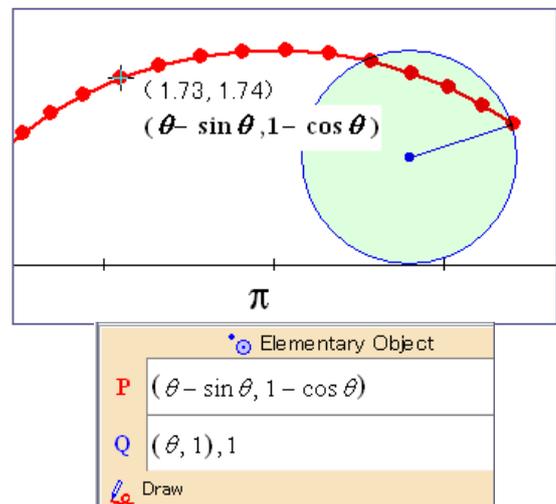
2. Move the cursor over the graph.

The graph equation is displayed when the cursor is moved over the graph.



- Displaying Locus Coordinates and Equations

The coordinates and equation are displayed with basic graphic elements and locii even if the cursor is moved over the image or curve.



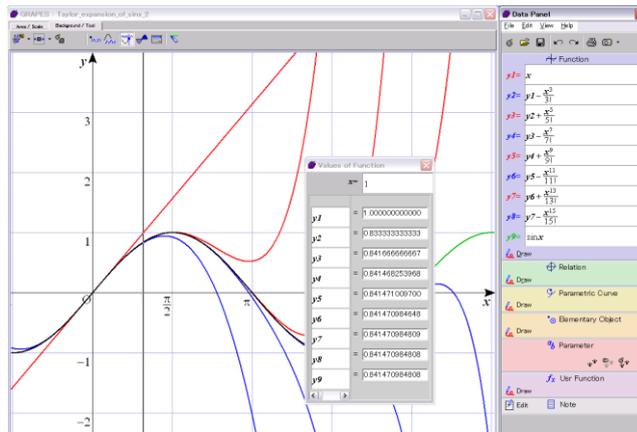
13-2 The Function Values Window

The function value for the x value in functions and any desired function equation is displayed in the Function Values Window, allowing multiple function values to be compared while viewing graphs.

- Displaying the Function Values Window

1. Click on the  (Show Value of Function) button on the Tools Pallet.

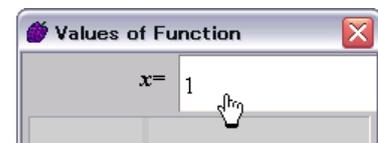
The Function Values Window is displayed. The perpendicular line indicating the x value is displayed in the Graph Window.



- Changing the x Value (calculator input)

1. Click on the x Value Window.
2. Enter with the Scientific Calculator.

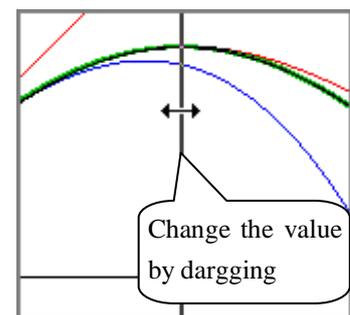
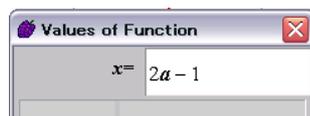
☆ When x is provided as a numerical value, the x value may be changed by dragging with the mouse as shown below.



- Changing the x Value (drag with mouse)

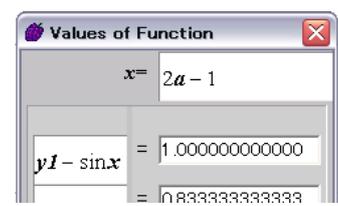
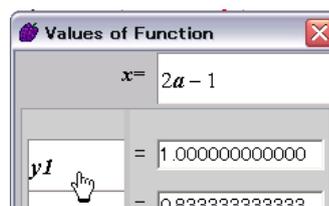
Drag the perpendicular line in the Graph Window.

☆ As shown below, when x is provided as an equation, it can not be moved by dragging with the mouse.



- Changing the Function Equation

1. Click on the function equation display section.
2. Enter the equation with the Scientific Calculator.



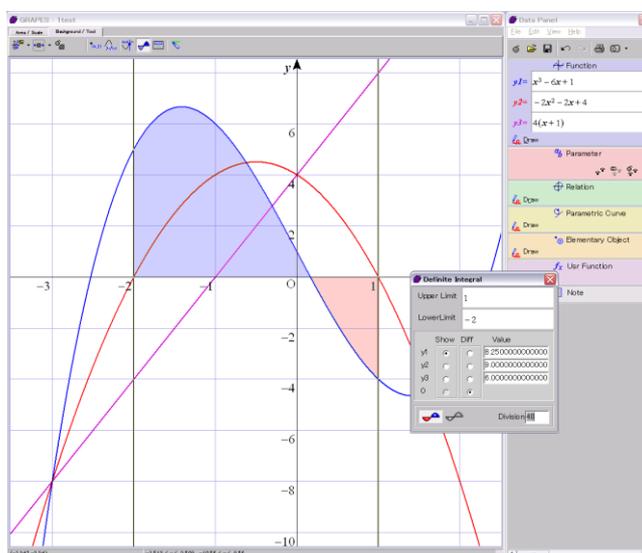
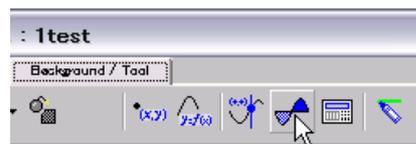
13-3 The Definite Integral Window

Definite integral values for functions are displayed in the Definite Integral Window. The integration area is displayed on the graph.

- Displaying the Function Values Window

Click on the  (Show Values of Definite Integral) button on the Tools Pallet.

The Definite Integral Window is displayed. Two perpendicular lines indicating the range of integration are displayed in the Graph Window.

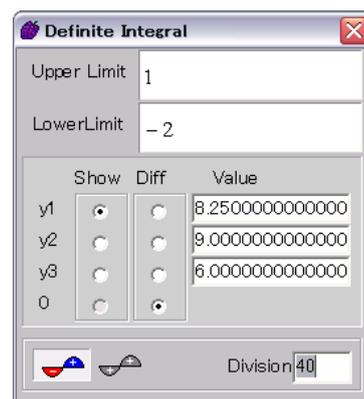


- Definite Integral Window

The Definite Integral Window displays the range of the definite integral, the [Show] button, the [Diff] button, and the definite integral values. The [Select Integrate/Area] button and the [Division] box are displayed at the bottom of the window.

The integration areas are displayed in blue for positive, and red for negative.

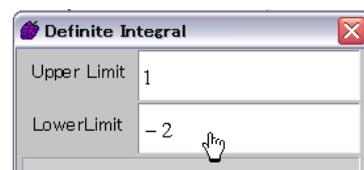
The parameters may be manipulated (e.g. increased or decreased) while the Definite Integral Window is displayed.



- Changing the Top and Bottom Limit Values (calculator input)

1. Click on the box displaying the top and bottom values.
2. Enter with the Scientific Calculator.

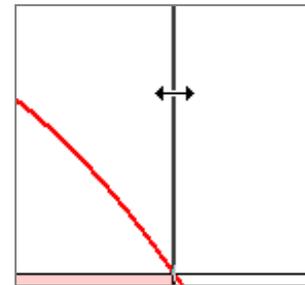
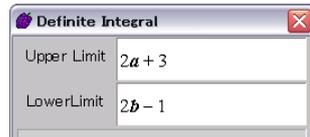
☆ When the top and bottom limits are provided as numerical values, change these values by dragging with the mouse as described.



- Changing the Top and Bottom Values (drag with mouse)

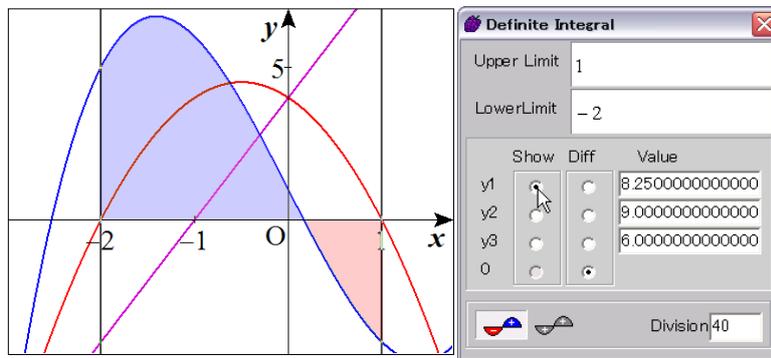
Drag the perpendicular line in the Graph Window.

☆ Drag the perpendicular line with the mouse as shown below, when the top and bottom limits are provided as equations.



- Displaying the Integration Area

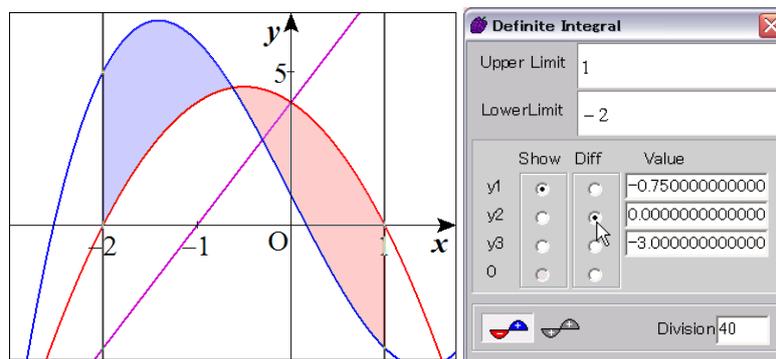
Click on the button inside the [Show] border for the function to be displayed.



- Displaying the Difference

Click on the button enclosed in the [Diff] border for functions for which the difference is to be taken.

The integral of the difference with the function selected here is displayed for all functions. Place a check in the bottom-most [Diff] button when the difference is not to be displayed.



- Area and the Definite Integral

Click on the  button (Definite Integral) to find the value of the definite integral.

Click on the  button (Area) to integrate with the absolute value applied to the function value.



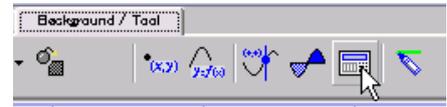
- Number of Partitions for Approximations

Definite integrals are calculated with a special 4th order approximation equation. No errors occur with up to and including 9th order polynomial expressions. Beyond this, the integration space is finely divided for calculation. The number of partitions is a natural number of 200 or less.

13-4 Entering and Calculating Functions With the Scientific Calculator

- The Scientific Calculator Display

Click on the Scientific Calculator button on the Tools Palette.



- Finding the Value of the Equation

Click on the  button (Calculate Equation).

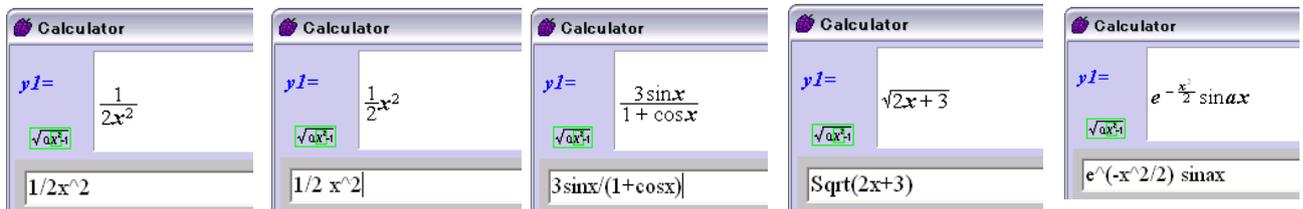
The value is displayed.

The equation may be a vector.



- Entering an Equation

The following shows entry of a number of typical equations.

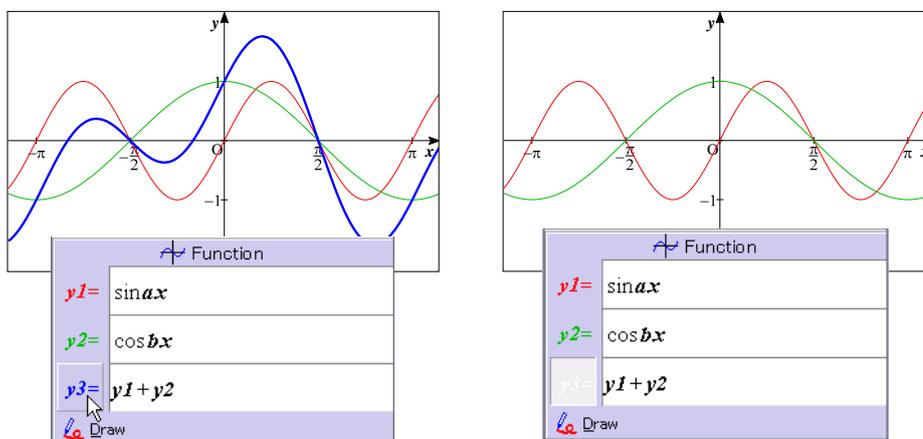


13-5 The Display Switch

Simply clicking on the Data Panel label allows instantaneous switching between display and non-display for any desired graph.

- Graph Display and Non-display

Click on the graph label notation.



Setting a graph to non-display with the display switch to also sets the afterimage to non-display. Locuses are also set to non-display.

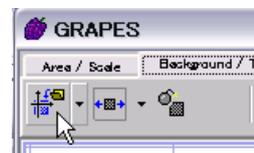
13-6 Using the Background

Wallpaper images may be applied to the graph display area background.

- Loading Background

Click on the  button (Paste Background).

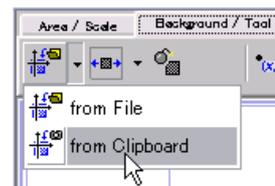
Select the image file. bmp, jpg, or gif image formats may be used.



- Applying the Background

Click on , and then click on the  button (Paste).

The image on the clipboard is applied to the background. Note that the image must always be copied to the clipboard first.

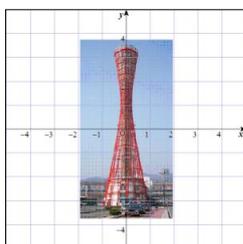
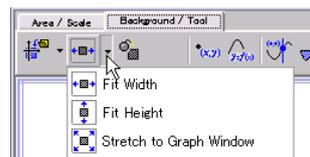
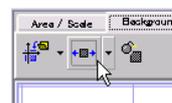


- Stretching the Background

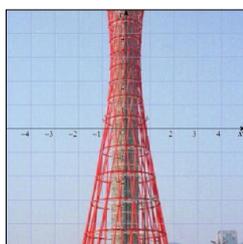
1. Click on the  button (Stretch Background Image).

Click again to undo the stretch.

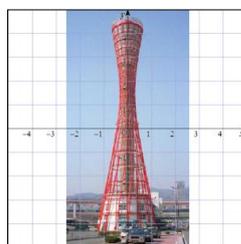
Clicking on  provides three types of stretch for selection.



Unchanged



 Fit width  Fit height



 Stretch to Graph Window



- Clearing the Background

Click on the  button (Clear Background).

- Saving the Background

When saving GRAPES data, only the loaded background file name is saved. The background is therefore set when loaded only if the GRAPES save file and the background file are in the same folder.

Background applied from the clipboard is not saved.

Chapter 14 Scripts

14-1 What is a Script?

Scripts are small programs which run within GRAPES.

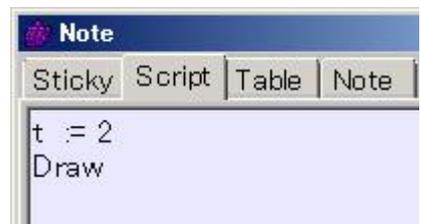
● Script Characteristics

- A control structure including repetitions and subscript calls.
- Parameter control.
- Substitution in graphic elements (point coordinates).
- Substitution in table cells.
- Calculation of various types of equations.
- Control of drawing, display, and images.
- Output of audio waveforms in accordance with function graphs.
- Setting of conditions for automatic execution.

● Script Editing and the Script Button

1. Click on [Edit] in the Notes Area.
2. Click on the [Script] tab.
3. Enter the script.
4. Click on [OK].

A button for the first line of the script appears in the Notes Area. This button is referred to as the Script button.



● Displaying the Script

- The first line is displayed as the Script button.
- When the first line includes a comment (//), the comment becomes the button title.
- The script is not displayed if the first line includes the HideScript command.



● Writing Scripts

- Only one command may be entered per line.
- Statements in the script are separated by a half-width space character.
- Comments are preceded by a double-slash (//).
- Statements used only in scripts (e.g. Draw) may be in upper or lower case.
- Parameters are entered in lower case. Functions are entered as is (e.g. sin, Sqrt, Pi).
- Multiple scripts (maximum of 30) are separated by blank lines.

14-2 Script Configuration Elements

- Variables

In addition to parameters and basic graphic elements, x , y , z , and w are available as local variables.

- Substitution

Equation values may be substituted in parameters, basic graphic elements, and table cells.

- Conditional Statements and Control Structures

‘if – then’ and ‘if – then - else’ for execution only while a certain condition is satisfied, ‘for - next’ for repeated execution, and ‘while – wend’ for repeated execution only while a certain condition is satisfied.

- Graph Drawing Control

Draw and OverDraw for calculation and drawing of graphs, Calc for calculation of coordinates, and Refresh and RefreshSticker for updating displays and stickers.

- Drawing Color Control

SetColor for changing the color of points and function graphs, SetLineColor for changing the color of lines, and SetInnerColor for changing the paint color.

- Image Control

ClrAImg for deleting images, SetAImg for creating images, and AImgOn and AImgOff for controlling images.

- Display Control

ShowObj for displaying graphs, HideObj for setting graphs to non-display, and InvShowObj for switching graphs between display and non-display.

- Display Range Control

SetArea for changing the display range.

- Sound Output

Play, PlayAfter, and PlaySync for output of audio waveforms in accordance with function graphs, and SetSampleRate for changing the sampling frequency.

- Setting of Conditions for Automatic Execution

‘on – change’ for automatic execution of scripts when the specified function is rewritten, or when points or parameter values are changed.

- Execution Control

‘SetTme – Wait’ for setting wait time, Pause for pause until input, and ‘end’ for terminating execution.

- Execute Another Script

Another script may be executed with ‘Call’. Up to four arguments may be passed to the called script.

- Script Button Non-display

The Script button is not displayed if HideScript is placed at the start of the script.

14-3 Executing and Terminating Scripts

- Execution With the Script Button

Click on the Script button to execute the script.



- ☆ An error message is displayed when the Notes Window is closed if the script contains a syntax error. Note that checking for syntax errors is not comprehensive. An error message may therefore not be displayed in the event of a syntax error.

- ☆ When an error occurs at runtime, the line containing the error is not executed and processing moves to the next line.

- Execute With the On change Instruction

When the first or second lines of the script contain the On Change command, the script is executed when the specified element is changed.

Example: ‘On a, b Change’ is executed when parameter *a* or *b* is changed.

- Terminate Script

Continue pressing the F1 key.

Used to interrupt and terminate scripts requiring considerable time for execution before processing is completed.

14-4 Variables and Substitution

- Global Variables

May be used as variables within parameters, point coordinates, table cells, and scripts.

- Local Variables

Four local variables (x, y, z, w) may be used. These variables are valid only within a script during execution. For example, variable x in script A and variable x in script B are unrelated.

Real numbers and error values may be substituted in local variables. Use X and Y when handling two-dimensional vector values with local variables. X is a variable set (x, y) and may therefore be written $X = (x, y)$. Similarly, $Y = (z, w)$.

Example: $X = (1, 2)$

and

$x := 1$

$y := 2$

return the same result.

The initial value of a local variable is an error value which is discarded after execution. When a script is called with the CALL command during execution of another script, the argument for the CALL statement is substituted in these variables.

Example: If the following is written in a C language-like format,

```
//Sum
```

```
  a := x+y
```

it appears as follows.

```
void Sum( x , y , z , w);
```

```
  { a = x + y }
```

- Substitution

Equation values may be substituted in parameters, point coordinates, table cells, and local variables x, y, z, and w as follows.

Substitution object := equation

Leave a space between the substitution object and ‘:=’.the same applies between ‘:=’ and the equation.

If the equation has no value (e.g. 1/0) when substituting in parameters and points (coordinate component), the substitution statement is not executed.

➤ Parameter := equation

Example: $a := \sin t$

- Point (i.e. the point coordinate component) := equation

Example: $P := (2,3)$, $P.x := 2$

☆ Speed of execution is slower with substitution in points.

For example, rather than using;

$P := (2,3)$

it is much faster to use $P(s,t)$

$s := 2$

$t := 3$

- $P(\text{point number}) := \text{equation}$

Point numbers are 1, 2, 3, ..., 7, 8, 9, ..., 21 for P, Q, R, ..., V, A, B, ..., N.

Example: $P(2) := (2,3)$ (same result as $Q := (2,3)$)

$P(n) := (2,3)$

- Local variable := equation

Example: $x := \tan t$

☆ In addition to real numbers, local variables may also take error values. The substitution statement is therefore executed even if the right of the substitution statement has no value, and an error value is substituted.

- $\text{Cells}(\text{row}, \text{column}) := \text{equation}$ (in addition to constants, equations may also be substituted in rows and columns)

Example: $\text{cells}(1,2) := 4 / a$

☆ If the value on the right is two-dimensional data, the value is substituted in the specified cell and the cell to its right.

Example: $\text{Cells}(1,2) := (4,5)$ has the same result as $\text{Cells}(1,2) := 4$, $\text{Cells}(1,3) := 5$.

☆ The cells in the first column are displayed if the column number is omitted.

- $\text{CellsP}(\text{row}, \text{column}) := \text{equation}$ (For row and column, you can enter an equation besides regular number)

When the right-hand side is vector data, the value is substituted in the specified cell and the cell to its right.

When the right-hand side is not vector data, the value is not substituted.

- ClrAllCells

Clears the values in all cells in the table.

- Inc and Dec

Inc/Dec increments/decrements the parameter or local variable value by the specified amount.

Inc(parameter, increment)

Dec(parameter, decrement)

When the increment/decrement is 1, the increment/decrement may be omitted (i.e. 'Inc(a)').

14-5 Control Structures and Conditional Statements

- for - next

The for - next statement repeats execution of the statements between 'for' and 'next'.

for 'parameter or local variable' := 'initial value' to 'final value' 'step'

 executable statement (multiple lines possible)

next

When 'step' is omitted, an increment of '1' is assumed.

- while - wend

Repeats execution of the statement between 'while' and 'wend' while the condition is satisfied.

while 'conditional expression'

 executable statement (multiple lines possible)

wend

- if - then -

Executes the command if the condition is satisfied.

 if 'conditional expression' then 'executable statement'

 As follows if commands extend over multiple lines.

if 'conditional expression' then

 executable statement (multiple lines possible)

end if

- if - then - else -

Executes the first command if the condition is satisfied, and executes a different command if the condition is not satisfied.

 if 'conditional expression' then 'executable statement1' else 'executable statement2'

 or

if 'conditional expression' then

 executable statement (multiple lines possible)

else

 executable statement (multiple lines possible)

endif

14-6 Drawing and Calculation

- Draw

Calculates the coordinates in accordance with the current parameter values, and draws the graph.

Add a colon to Draw (i.e. 'Draw:') to eliminate the locus.

The entire display, except for stickers, is updated.

- ☆ Substituting values in variables in a script does not result in recalculation of the graph. Draw and Calc are used for recalculation.

- OverDraw

Basically similar to Draw, except that a new graph is added to the screen, rather than the screen being erased and redrawn. Faster than Draw.

- Calc

Calculates the coordinates in accordance with the current parameter values, and saves the loci and image in internal memory. Note that the screen is not updated. Use Draw or Refresh (described below) to update the screen.

Add a colon to Calc (i.e., 'Calc:') to eliminate the locus.

Since the graph is not drawn, Calc is faster than Draw and OverDraw.

- Refresh

Calc leaves the image and results of loci calculation in internal memory, however the screen is not updated. Refresh displays the image and locii left in memory with Calc in the Graph Window.

Refresh displays the image and locii, however it does not display the current graph. Use Draw to display the current graph. Draw also displays the image.

- RefreshSticker

Updates sticker display.

The current result is updated when an equation value is displayed in a sticker.

14-7 Changing Graph Color

- SetColor(element name, color)

Changes the display color of a graphic element. Changes point color for basic graphic elements and curves, graph color for function graphs, and line color for linked graphic elements.

- Graphic elements may only be specified one-at-a-time.
- Colors are specified in a 24-bit integer value representing RGB, however since GRAPES cannot handle hexadecimal values, color is specified with the rgb and hsl functions.

Example: `SetColor(y1 , rgb(0.8, 0, 1))`

The function graph y1 is displayed in purple.

- SetLineColor(element name, color)

Changes the display color of a graphic element. Changes line color for basic graphic elements and curves, graph color for function graphs, and line color for linked graphic elements.

- SetInsideColor(element name, color)

Changes the paint color for basic graphic elements, curves, and linked graphic elements.

14-8 Deleting and Creating Afterimages

- ClrAImg(element name), ClrAImg

Deletes the image and locii of a graphic element.

Note that ClrAImg is an abbreviation of 'Clear After Images'.

- `ClrAImg(element name)` deletes only the image of the graphic element.
- Functions, relations, curves, and basic graphic elements, as well as linked graphic elements may be specified as the graphic element name. Multiple graphic elements are delineated with commas.

Example: `ClrAImg(y1,C2,P,L2)` deletes the images of the function y1, the relation C1, the graphic element P, and the linked graphic element L2.

- Use `ClrAImg` to delete all graphic element images without the need to specify graphic element names.

- SetAImg

Leaves images of all currently drawn graphs.

Inside colors of circles and rectangles are not left as images.

14-9 Display Control

- AImgOn and AImgOff

Use of the AImgOff command ensures that, irrespective of the setting for the graphic element, no image is recorded when the  button (Image OFF) in the Parameter Area is clicked. Returns to the original status with the AImgOn command.

- ShowObj(element name), HideObj(element name), InvShowObj(element name)

- ◆ HideObj(element name)

The graph is no longer displayed when the graphic element display switch is clicked.

Functions, relations, curves, and basic graphic elements, as well as linked graphic elements may be specified as the graphic element name. Multiple graphic elements are delineated with commas.

Example: HideObj(y1,C2,P,L2) does not display the function y1, the relation C1, the graphic element P, and the linked graphic element L2.

- ◆ ShowObj(element name)

Clears the clicked status of the graphic element display switch, and displays the graphic elements.

- ◆ InvShowObj(element name)

Toggles graphic element display/non-display.

- SetArea(x1,y1,x2,y2)

Displays the area having its diagonal between the two points (x1,y1) and (x2,y2) on the entire screen.

This command redraws all graphs.

14-10 Sound Output

- Play(parameter, start seconds, finish seconds, waveform function)

- ◆ Outputs sound digitized at a sampling rate of 22,050Hz.

Example: `Play(t , 0 , 2 , sin880Pit)` ('Pit' is the product of π and t) outputs a 440Hz sine wave for 2 seconds.

- ◆ Provides vector data at stereo output.

Example: `Play(t , 0 , 2 , (sin 880Pit , sin 882Pit))` outputs a 440Hz sound from the left channel and a 441Hz sound from the right channel.

All parameters used with GRAPES, and the variable x , may be used as parameters.

The performance begins after all data has been calculated. Functions may also be used as waveform functions.

- Play

The Play command requires no parameters, and plays the data remaining in the work memory.

- ◆ Plays Wave data created with the PlayAfter command.
- ◆ Replays the Wave data played last.

- PlayAfter(parameter, start seconds, finish seconds, waveform function)

- Wave data created with PlayAfter is placed in the work memory and is not played.
- When the next PlayAfter command is executed while Wave data remains in the work memory, the sets of data are joined.
- The Play command is executed to play Wave data created in the work memory.
- The play order may become confused in high performance PCs if multiple Play() commands are executed in succession. In this case, create connected data with the PlayAfter command, and play it with the Play command.

Example: `PlayAfter(t , 0 , 1 , sin400Pit)`

`PlayAfter(t , 0 , 2 , sin600Pit)`

`Play`

Output a 200Hz sine wave for 1 second, and a 300Hz sine wave for 2 seconds.

- PlaySync(parameter, start seconds, finish seconds, waveform function)

The PlaySync command differs in only one point from the Play command.

- Play with the Play command is conducted in the background, however with PlaySync, the next command is not executed until play is finished.

- SetSampleRate(sampling rate)

Changes the sampling rate.

Example: Set to 48,000Hz as follows.

SetSampleRate(48,000)

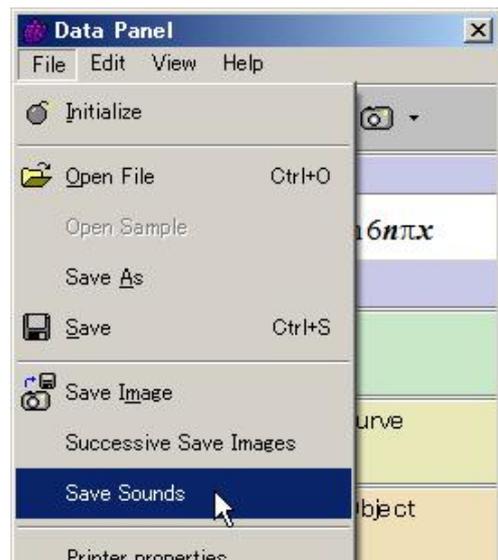
The new sampling rate is retained until GRAPES is terminated.

- A set of Wave data may be played for a maximum time of 480,000/sampling rate (seconds).
- Digitization is fixed at 16-bit. Since the interval -10 – 10 is mapped to -32768 – 32767 in GRAPES, the sound becomes distorted if the waveform function value exceeds this range.

- Saving Sound

The last played sound is saved to a Wave file.

Click on [Save Sounds] in the [File] menu.



14-11 Execution Control

- SetTime(milliseconds) - Wait

SetTime sets a timer. Waits at the Wait position until the set time has elapsed. Processing moves immediately to the next command when the set time has elapsed. Specified in milliseconds.

Example: SetTime(n)

a := 3

Draw

Wait //Processing waits n milliseconds after execution of SetTime.

- Wait (milliseconds)

Halts processing for the specified time. Specified in milliseconds.

- Pause, Pause(message)

Halts execution of a script and waits for input.

The message is displayed and processing is halted when Pause(message) is used.



Pause



Pause(Check point movement)

Click on [OK] to continue or [Cancel] to halt processing.

- Call(subscript name, argument 1, argument 2, argument 3, argument 4)

Executes another script in the same project. Control moves to the next line following execution.

The values of the four arguments of the Call command become the initial values of the subscript local variables x, y, z, and w. All of these arguments, or consecutive arguments (e.g. last 3, 2, or 1 arguments), may be omitted. Error values are substituted in omitted arguments.

Arguments must contain real numbers or error values. When a two-dimensional vector value is assigned to an argument, it is assumed that two real number values have been assigned. For example, all of the following have the same result.

Call(test , (1,2),(3,4)), Call(test , (1,2),3,4), Call(test , 1,2,(3,4)), Call(test , 1,2,3,4)

Note that the following format is not permitted.

```
Call(test , 1,(2,3),4)
```

- End

Terminates execution of a script. If the script being executed has been called from another script with the Call command, control is returned to the calling script.

- on element name change

The script is executed automatically when the specified element is changed.

- Parameters, basic graphic elements and curves (e.g., P, Q), defined functions, functions, and relations may be specified. Linked graphic elements may not be specified. Multiple elements are delineated by commas.
- This command must be entered in the first or second line of the script.

Example: on a,b,f change

Executed when parameter a or b, or function f, is changed.

In the following example, when the value of parameter a is changed, parameter t is initialized, and the image and locii are deleted.

```
//Initialize  
on a change  
t := 0  
calc  
ClrAImg
```

- ◆ Scripts and Deletion of Afterimages

Afterimages are deleted after initializing parameter values. This is due to the fact that, since images are recorded immediately before a new graph is drawn, the initial image cannot be deleted before the image for the current graph created immediately before the next graph is drawn.

- ◆ Execution Timing

Automatic execution with the on – change statement occurs only when the target element is changed directly from the Data Panel. When an element is changed from the Data Panel, coordinates are recalculated and the graph is drawn, however automatically executed scripts are executed immediately after change from the Data Panel, and coordinates then recalculated and the graph drawn.

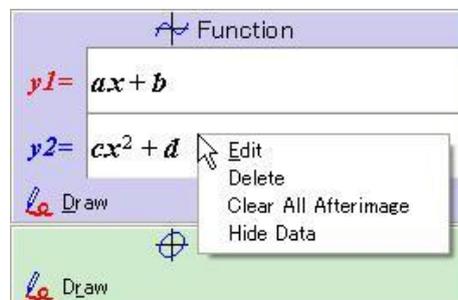
Chapter 15 Tips

15-1 Using Pop-up Menus

- Data Panel Pop-up Menu

Right-click on the function equation display section of the Data Panel.

The pop-up menu appears, and the function may be amended or deleted, or the image deleted.



- Sticker Pop-up Menu

Right-click on the sticker.

The pop-up menu appears, and the entire sticker may be edited, set to non-display, or deleted.

This operation is applied to all stickers, even when multiple stickers are displayed on the screen. For example, all stickers are set to non-display if non-display is selected.

- Graph Display Area Pop-up Menu

Right-click the Graph Display Area.

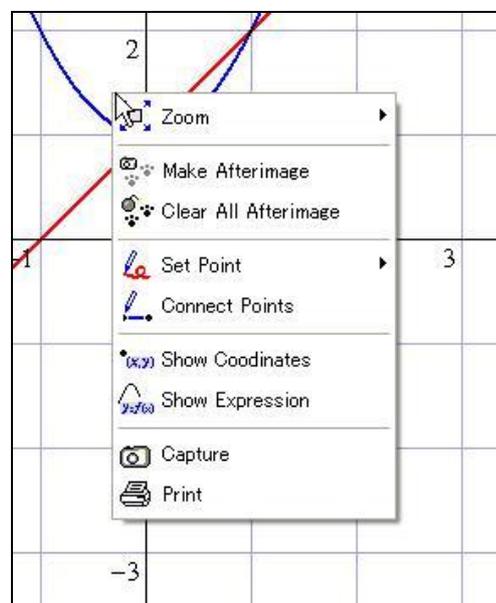
The pop-up menu appears, and various operations may be applied to the graph.

- ◆ Zoom
 - × 10, × 5, × 2, × 1/2, × 1/5, × 1/10

- ◆ Image operations
- ◆ Set a point
- ◆ Graph property display

The properties of the graph or graphic element at the clicked position are displayed and may be edited.

- ◆ Show of graph coordinates and expression
- ◆ Capture and Print



15-2 Printing and Pasting Screens

● Printing Screens

1. Click on the  (Print) button on the Toolbar.

The Print Screen Setup dialog appears.

2. Adjust the screen size.

Click on [Printer properties] under [Size of Paper] when changing printer settings.

3. Select the print settings.

When [Graph (hi-res Color)] or [Graph (hi-res B/W)] are selected, vector data is used to print smooth graphs.

When [Graph (capture)] is selected, the graph is printed at the screen quality. When [Whole GRAPES] is selected, both the Graph Window and the Data Panel are printed. Place a check in the [Adjust Print Size] checkbox under [Size of Graph] for high-quality printing.

4. Click on [OK].

● Copying the Screen and Pasting Into a Word Processor Document

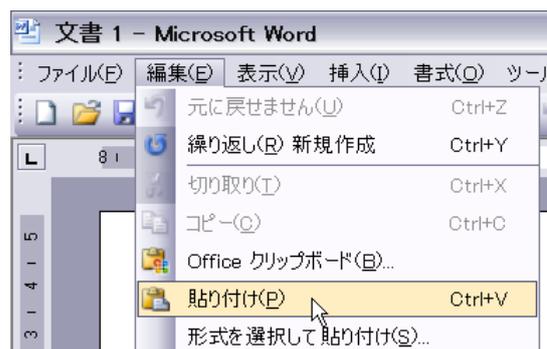
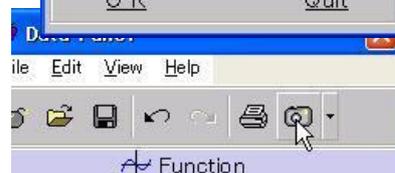
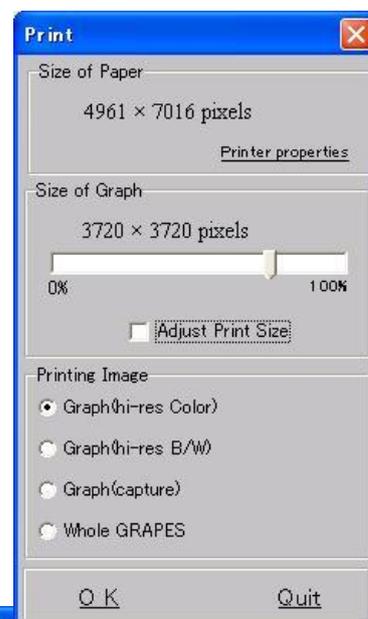
1. Click on the  (Capture) button on the Data Panel.

The entire Graph Display Area is copied to the clipboard. Click on  to the right of the Copy button to select the data format and copy target.

2. Click on the [Paste] in the [Edit] menu of the word processor.

The image data on the clipboard is pasted into the document. See the word processor manual for details on subsequent operations.

- ☆ Depending on the software used in pasting to the document when copying the screen, it may not be possible to obtain sufficient screen resolution despite selecting [Graph (hi-res Color)] or [Graph (hi-res B/W)]. In this case, click on the Copy key while holding down the Shift key.

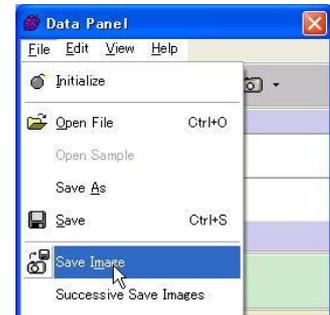


15-3 Saving Screens and Sound

- Saving the Graph Screen

Click on [Save Image] in the [File] menu.

Screens may be saved in Windows Bitmap (*.bmp), JPEG (*.jpg), Enhanced Metafile (*.emf), and Windows Metafile (*.wmf) formats. Enhanced Metafile and Windows Metafile formats permit saving of high-quality graphs without jagged edges.

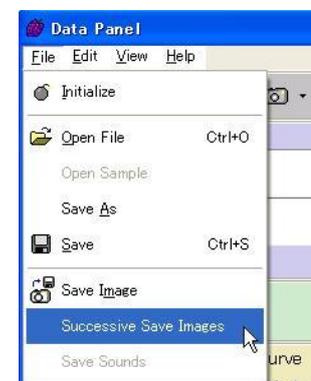
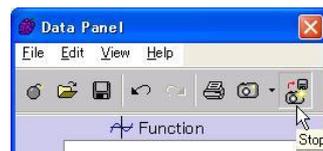


- Continuously Saving the Graph Screen

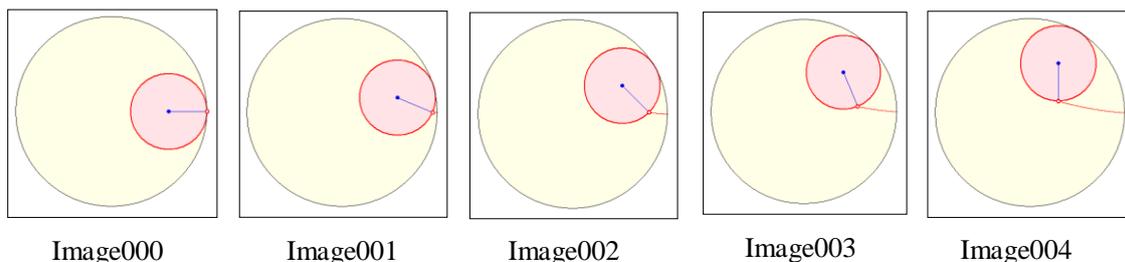
1. Click on [Successive Save Images] in the [File] menu.

The [Successive Save] icon  appears on the Toolbar, and the screen image is then saved each time the screen is updated.

2. Click on the  (Successive Save) icon.



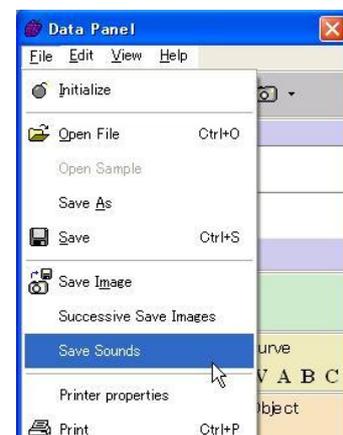
The following example uses the file name 'Image' for continuous save. The screen image is saved each time parameters are changed. The text string beneath each screen image is the file name under which each is saved.



- Saving Sound

Saves the sound last played with the Play command in the script.

Click on [Save Sounds] in the [File] menu.

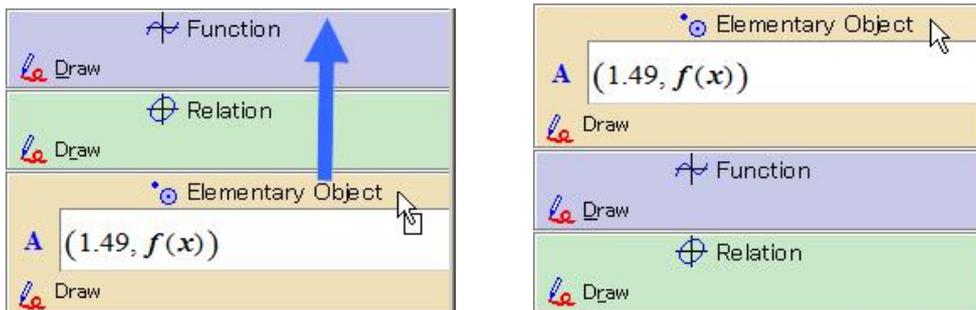


15-4 Customizing the Data Panel

The display sequence, and display/non-display, may be changed for each area of the Data Panel and for each individual item of data.

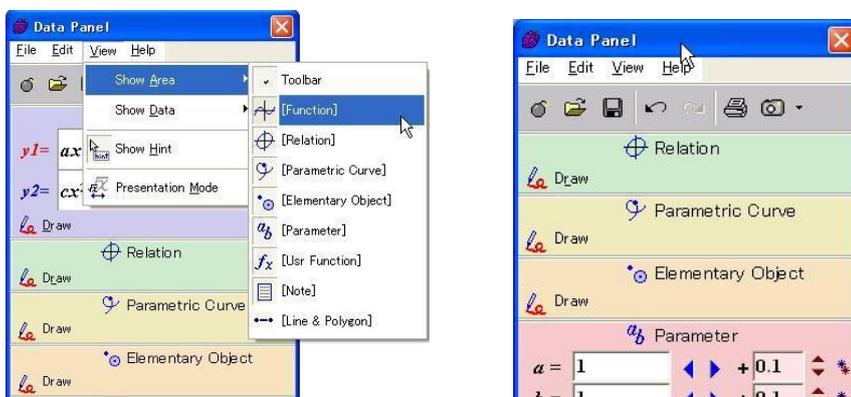
- Moving Each Area

Drag the area title part to another area.



- Hiding Each Area

Select [Show Area] in the [View] menu, and click on the display/non-display area.

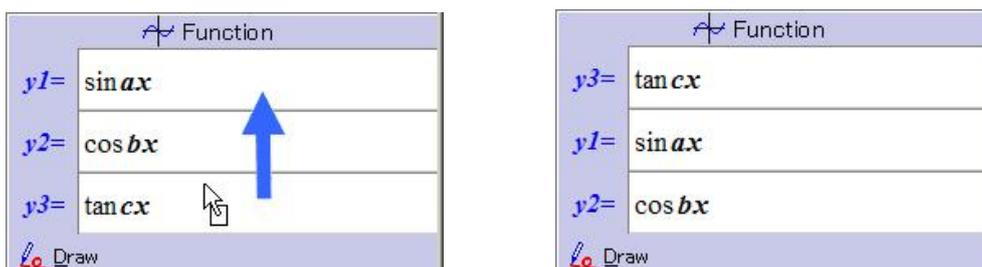


- Moving Individual Equations and Data

Drag the Function Equation Window and Parameter Value Window up or down.

The data display Area sequence may be changed within the same area.

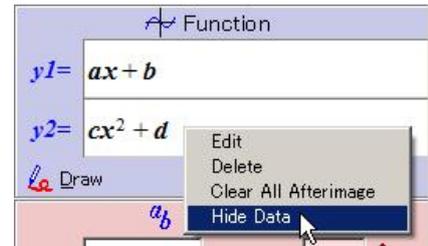
The linked graphic element and sticker display sequences can not be changed.



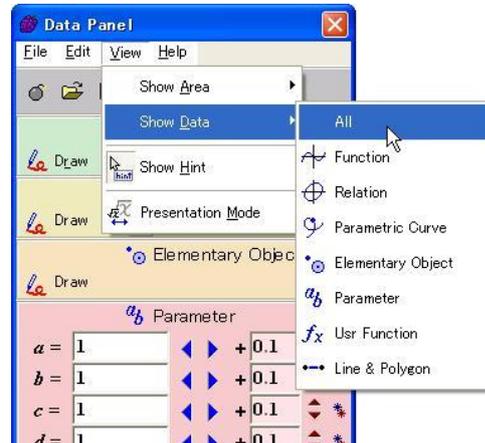
- Hiding Individual Equations and Data

Right-click on the data to be hidden, and select [Hide Data].

Functions, graphic elements, and parameters may be hidden.



To re-display hidden data, select [Show Data] in the [View] menu.

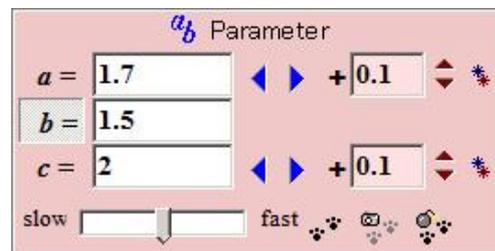
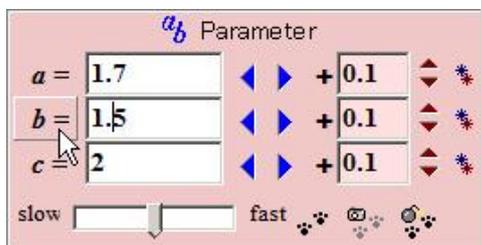


- Fixing Parameter Values

Click on the parameter notation.

A parameter value cannot be changed while the label is selected.

Click again to free the value.



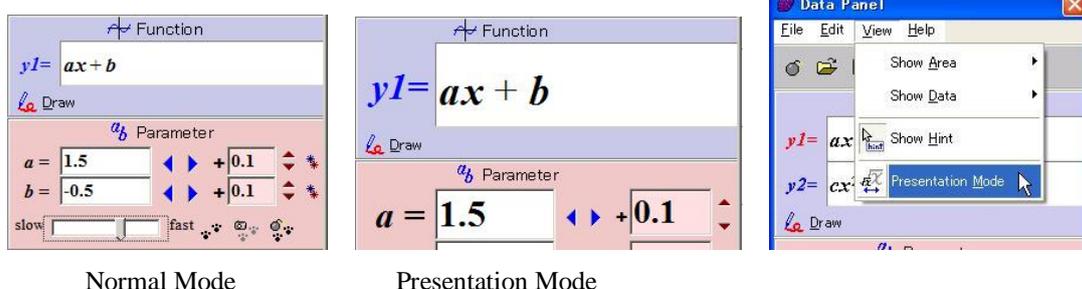
15-5 Use in Presentations

- Presentation Mode

Displays equation images and parameter values in double-sized font.

Click on [Presentation Mode] in the [View] menu.

Click again to return to the original font size.



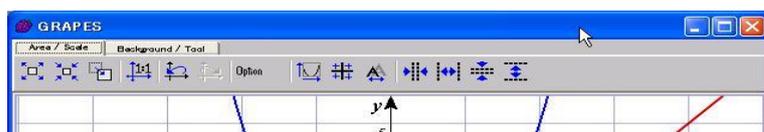
- Enlarging the Graph Display Area

Hide the Control Pallet, and enlarge the Graph Display Area.

Right-click on the Control Pallet, and click on [Auto-Hide ControlPalette].



Move the cursor to the top of the Graph Display Area to display the Control Pallet.



- Using Markers

1. Click on the  (marking Pen) button on the Tools Pallet.

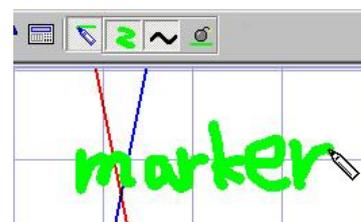
The Marker Pallet appears, and the cursor assumes the shape of a marker.

2. Draw by dragging with the left mouse button.

Drawing continues while the left button is held down.

3. Delete the marker by dragging with the right mouse button.

4. Click on the  (Delete All Markers) button to delete all markers.



15-6 GRAPES Specification

- Specification

- Functions 20
- Relations 9
- Basic graphic elements and curves 21 (excluding origin)
- Curve apexes Maximum 5000
- Defined functions 8
- Linked graphic elements 30
- Parameters 14
- Stickers (partition display) 10
- Scripts 20
- Tables 200 x 10 cells
- Graph display size Maximum 1600 x 1600 pixels
- Afterimages 3000
- Undo (functions) 50 actions
- Undo (areas) 200 actions